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SIMULTANEOUS MEASUREMENT OF THE CAPACITY AND POWER FACTOR OF CONDENSERS.

By Frederick W. Grover.

1. INTRODUCTION.

In a perfect condenser—that is, a condenser without absorption or leakage—the phase of the current is 90° ahead of that of the impressed electromotive force. Although many condensers closely approximate to this ideal case, it is only with well-insulated air condensers that the angle of advance may be regarded as sensibly 90°. In condensers having for a dielectric paper filled with paraffine or beeswax, and even with mica condensers, there is an appreciable energy component of the current in phase with the electromotive force.

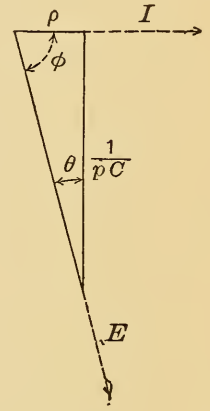

Fig. 1.

In what follows the small angle θ, by which the phase angle ϕ falls short of 90°, will be called for convenience the *phase difference* of the condenser. The power factor of the condenser is equal to $\cos \phi = \sin \theta$.

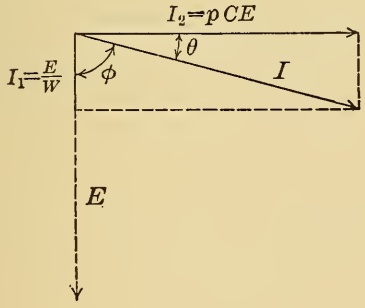

Fig. 2.

A condenser having absorption is equivalent, therefore, in its effect on the phase of the current, to a capacity C in series with a resistance ρ, Fig. 1, of such a value that

$$\tan \theta = pC\rho \qquad (1)$$

(where $p = 2\pi$ times the frequency) or to a capacity C in parallel with a resistance W, Fig. 2, such that

$$\tan \theta = \frac{1}{pCW} \qquad (2)$$

Since θ is usually a very small angle, it is evident that for ordinary frequencies ρ is a small resistance, while W is a very large one. We shall sometimes use one of these conventions and sometimes the other, according as one or the other gives greater simplicity to the formulæ.

The power factor of a condenser may be determined—

1. By measuring the energy loss and computing θ.
2. By measuring the angle θ directly.

Since the energy loss in a condenser is very small, it is difficult to obtain accurate results by the first method. Such determinations have, however, been successfully made.

By the use of two methods in which the condenser to be tested was made part of a resonance circuit, Rosa and Smith[1] were able to measure the loss in the condenser with a wattmeter. The mean power factor of six condensers having a dielectric of beeswax and rosin was found to be about 0.02 at 30°. This corresponds to a value of θ of about 1°.

The same observers[2] measured directly, by a continuous-flow calorimeter, the amount of heat generated in these same condensers, and obtained results in good agreement with those by the wattmeter methods.

Steinmetz[3], using one of the wattmeter methods employed by Rosa and Smith, found values for the power factors of some General Electric Company paraffined paper condensers in the neighborhood of ½ per cent, the result obtained being dependent on the frequency. For these condensers, therefore, θ was of the order of magnitude of 0° 15′.

It is, however, less difficult to measure the phase difference than the energy expended in the dielectric. By means of one of the Rowland electrodynamometer methods, Potts[4] measured the increase in effective resistance which was produced by introducing a condenser in series with a bridge arm which already contained an inductance and the hanging coil of the electrodynamometer. To calculate the angle θ it is necessary to know, in addition to this, not

[1] Phys. Rev., Jan., 1899.
[2] Phys. Rev., Feb., 1899.
[3] Lond. Electrician, July 5, 1901.
[4] American. Jour. Sci. (4) **10**, p. 91; 1900. Phys. Zs. **2**, p. 301; 1901.

only the steady current resistance of the coils and their leads, but also the change of this resistance, with the frequency. Further, the unavoidable temperature changes of the copper wire of the coils requires that their resistances be frequently determined.

Rosa[5] has developed a number of dynamometer methods for the measurement of θ, and has shown that they give results in good agreement with one another, and with those he found by the calorimetric method.

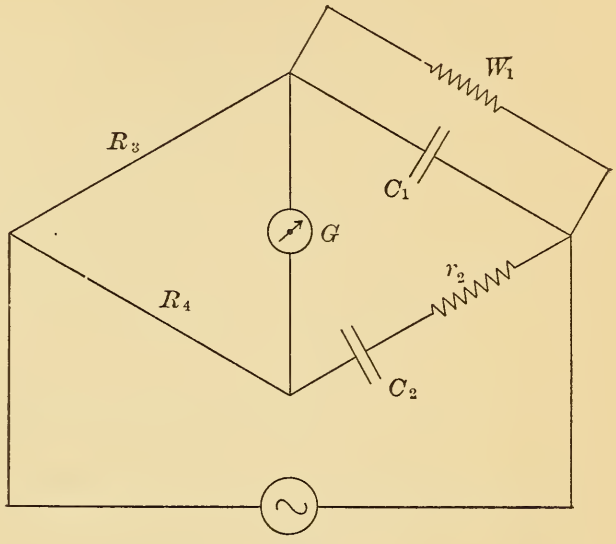

Fig. 3.

In 1891, Max Wien gave[6] several alternating current bridge methods for the measurement of inductances and capacities. Among these is an arrangement for comparing the capacities of two condensers and simultaneously obtaining the absorption of the condenser C_1, Fig. 3, the absorption of C_2 being assumed equal to zero. He assumed the absorption of C_1 to be represented by a resistance W_1 in parallel with the condenser. The bridge is balanced by alternately adjusting r_2 and one of the ratio arms. When the current in the tuned galvanometer G is zero

$$\frac{C_1}{C_2} = \frac{R_4}{R_3} - \frac{r_2}{W_1} \qquad \text{(A)}$$

$$p^2 C_1 C_2 W_1 r_2 = 1 \qquad \text{(B)}$$

(Wien's nomenclature is here changed to agree with that of this article.)

He also shows how this method may be used to determine *two capacities simultaneously*. To do this, a resistance r_1 is inserted in parallel with C_1, such that, to a first approximation, the resistance

[5] This Bulletin, **1**, p. 383; 1905.
[6] Wied. Annalen, **44**, p. 681; 1891.

W_1, due to the absorption of the condenser, may be neglected. To correct for the absorption of C_1, a second reading is taken with the resistance r_1 removed. In an example given by Wien the error due to the absorption of C_2 was neglected.

The first-mentioned arrangement of this method has since been used by Hanauer [7], who measured the change of the capacity and the resistance W_1, with change of frequency, for a number of solid and liquid dielectrics. The few results obtained for mica gave values of θ as large as several degrees.

This method in a modified form has been used by the author since 1904.

Since the work in this paper was begun Monasch has published[8] the results of an elaborate investigation (using Wien's method) of the energy losses in cables. Pressures of about 1,500 volts were ordinarily used, although occasionally values as high as 9,000 volts were employed.

In this very interesting paper Monasch describes his variable standard air condenser, constructed to withstand the high voltages used, and discusses the effect of errors due to the capacity of the resistances and to the electrostatic capacity of the bridge with respect to its surroundings. The former of these difficulties he met by the use of resistances coils wound in the manner suggested by Chaperon, and he gives measurements to show that the effect of the latter was in his apparatus very small. The cables tested had a capacity of a few thousandths of a microfarad.

The present paper deals particularly with the determination of the power factor of condensers to be used for precision measurements of inductance and capacity. Especial attention has been paid to the elimination of constant errors, and the different methods have been compared with one another to ascertain how well this has been accomplished.

Examples will be given to show that the power factor gives a good idea of the quality of the condenser, the order of magnitude of its residual charges, and the change of capacity with the frequency and method of charge, all of which depend upon the absorption. It

[7] Wied. Annalen, **65**, p. 789; 1898.

[8] Inaugural Dissertation, Dantzig, 1906, and Annalen der Phys. **22**, p. 905, 1907.

is the best single test which can be made, and has the great advantage that the power factor can be determined simultaneously with the capacity.

2. THE COMPARISON OF CAPACITIES.

The arrangement shown in Fig. 4 has long been used with steady currents, with intermittent currents (using a rotating commutator in the battery and galvanometer circuits), or with alternating currents. In the latter case a telephone has frequently been employed as a detector of current in the galvanometer arm AB. As it is generally impossible, even with a sine wave, to make the current in this arm zero, it has been customary to determine that ratio of R_3 and R_4 which will bring the current to a minimum. The capacity ratio is then determined from the equation

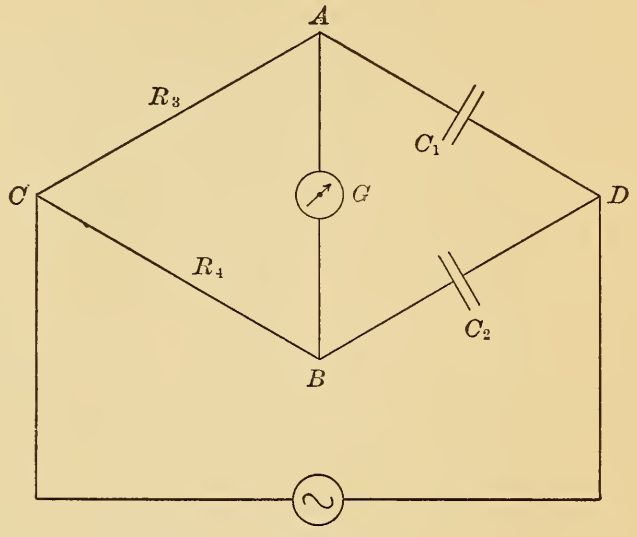

Fig. 4.

$$\frac{C_1}{C_2}=\frac{R_4}{R_3} \tag{3}$$

When the absorption of the two condensers is nearly the same, the sound in the telephone can be made nearly to disappear, and fairly good settings may be obtained; but, with such differences in the power factor of the two condensers as often occur, the minimum is by no means small, and its position is far from being sharp. This is illustrated in the following example. Suppose

$C_1 =$ 1 mf $\quad C_2 =$ 1.05 mf
$\theta_1 =$ 30′ $\quad \theta_2 =$ 1′
$R_4 =$ 1,000 ohms. $\quad p =$ 2 π times the frequency $= 600$
$E =$ 100 volts = emf. impressed on the bridge
$g =$ 200 ohms = resistance of galvanometer circuit.

This is the case of the comparison of a rather poor paper condenser with a good mica condenser.

From these data we find by equation 1,

$$\rho_1 = 14.545 \text{ ohms}$$
$$\rho_2 = 0.462 \text{ ohms}$$

and it is not difficult, though rather tedious, to calculate the galvanometer current for different values of R_3. If the condensers were perfect the bridge should be balanced with $R_3 = 1{,}050$ ohms (equation 3). The following results were obtained:

R_3	Current in galvanometer arm	R_3	Current in galvanometer arm
Ohms	*Micro-ampères*	*Ohms*	*Micro-ampères*
1,045.0	20.184	1,050.1	14.324
1,049.0	14.506	1,050.25	14.333
1,049.75	14.336	1,051.0	14.502
1,049.9	14.325	1,055.0	18.870
1,050.0	14.323		

This table shows that the value of R_3, corresponding to minimum current, is substantially the same as it would be if the condensers were perfect. The minimum is, however, so "flat" that with an ordinary telephone where 1 micro-ampère can be detected it is doubtful if settings could be made closer than to the nearest ohm; and, if the wave form of the supply deviates very much from a sine curve, the settings would be still worse. If, instead of a telephone, we use a vibration galvanometer it will still be difficult to make close measurements. The vibration galvanometer used in these experiments, which responds to the frequency assumed in this example, will detect 0.2 of a micro-ampère. On the assumption that this corresponds to a deflection of 0.1 mm, the deflection produced by the minimum current in the example would be about 7.1 mm; and, as it would probably be impossible to detect a change in this of less than 0.1 mm, a difference of 1 ohm in R_3 on either side of the balance point would just begin to have a noticeable effect on the galvanometer current. It is, therefore, doubtful if the balance point could be estimated much closer than to the nearest half ohm—that is, to 5 parts in 10,000.

The reason why it is impossible to balance the bridge shown in Fig. 4 is evident from the following considerations. In order that the current in the arm AB, Fig. 4, shall, at every moment, be zero, it is necessary that the potentials of points A and B shall always be the same. This requires that the potential differences AC and BC must not only be equal, but must have the same phase. The same is true of the potential differences AD and BD. In other words, the currents in the two sides of the bridge must have the same phase.

Fig. 5 shows the vector diagrams of the two sides of a bridge containing condensers without absorption or leakage, the galvanometer being removed, and the voltage impressed on the bridge being CD.

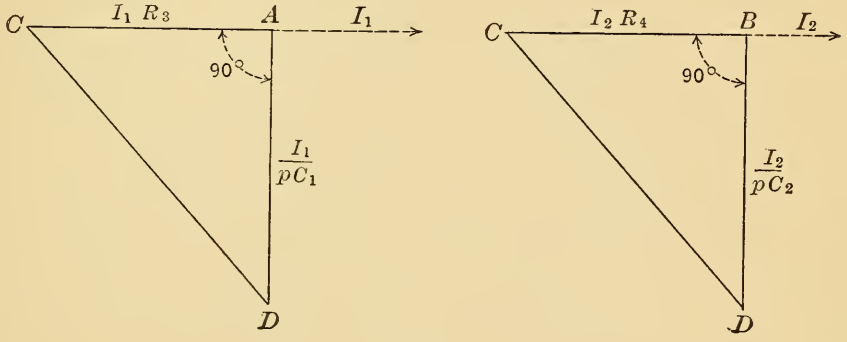

Fig. 5.

In order that A and B shall always be at the same potential at the same moment, triangles ACD and BCD must be equal. CD being common to both, this condition is satisfied if

$$I_1 R_3 = I_2 R_4$$

$$\frac{I_1}{pC_1} = \frac{I_2}{pC_2}$$

$$\therefore pC_2 I_1 = pC_1 I_2 \text{ and } \frac{C_2}{C_1} = \frac{I_2}{I_1} = \frac{R_3}{R_4}$$

as in equation 3.

If, however, we are comparing condensers which have an appreciable absorption, the angles CAD and CBD, Fig. 5, will no longer be right angles, as in the preceding case, and will not be equal unless the condensers have equal power factors. Such a case is illustrated in Fig. 6. The triangles EAD and FBD are the phase diagrams of the condensers C_1 and C_2, respectively, their

absorptions being represented by the fictitious resistances ρ_1 and ρ_2. By equation (1), their power factors are

$$\cos \text{EAD} = \sin \theta_1$$
$$\cos \text{FBD} = \sin \theta_2.$$

Since these angles depend on the absorption of the condensers, it will be impossible, simply by adjustment of the ratio $R_3 : R_4$, to make the triangles CAD and CBD equal. That is, it is impossible to

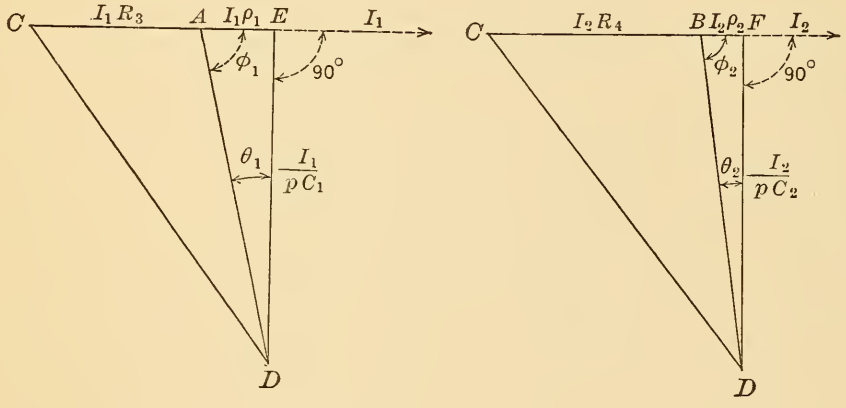

Fig. 6.

make the potentials of A and B the same. It is only possible, then, to find a value of the ratio $R_3 : R_4$ which will make the galvanometer current a minimum. Therefore, in order to bring the points A and B to the same potential it is necessary to shift the phases of the condenser currents. This can be done by any one of three methods.

3. SERIES RESISTANCE METHOD (WIEN).

An adjustable resistance r is inserted in the arm with the condenser having the smaller power factor, and the bridge is balanced by alternately adjusting one of the resistances R_3, R_4, and the resistance r.

This is easily and quickly accomplished, if, with each adjustment of one of the two resistances, the observer systematically finds a value of the resistance which will make the galvanometer current a minimum. The successive minima thus obtained very rapidly approximate to zero.

This is illustrated in Fig. 7, in which, as in Fig. 6, the phase diagrams of C_1 and C_2 are represented by the triangles AED and BFD, respectively.

The galvanometer current i_g is proportional to the distance AB.

The locus *aa* of the point A, when R_3 is varied, is one of a family of circles, passing through C and D, and having for a parameter the angle EAD. We will call these *equal phase curves.* Similarly, with R_3 as a parameter and r as a variable, we may draw a second family of circles, which are, when angle EAD is nearly 90°, very closely orthogonal to the first and may be called *equal resistance curves.*

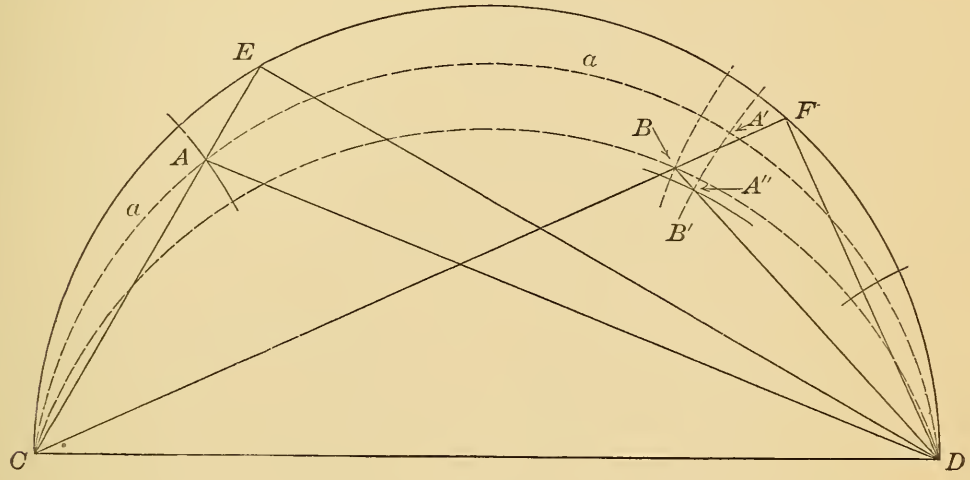

Fig. 7.

Starting from the point A, suppose the resistance R_3 is varied until the galvanometer current is a minimum. A will move on its equal phase curve *aa* to a point A′ which satisfies the condition that a line drawn through it, tangent to *aa*, is perpendicular to the line BA′. A′ will lie near, but not upon, the equal resistance curve which passes through B. When, now, r is varied till i_g is again a minimum, a point A″ is found, where the equal resistance curve A′B′ is tangent to a line perpendicular to BA″. Again varying R_3, a minimum value, BA‴, of the current i_g will be obtained, which is still smaller than the preceding minima BA′ and BA″, and thus the condition of zero current is rapidly approximated.

If, however, there be a component of the galvanometer current, due, for instance, to electrostatic charges on some part of the bridge, which does not depend on the capacities and power factors of the

arms, it will be impossible to bring the current i_g to zero by any adjustment of R_3 and r. Not only will this reduce the sensitiveness (see p. 376), but if the value of this disturbing current is affected by changes in R_3 or r, a minimum current may be obtained with values of these which differ appreciably from those corresponding to the capacity ratio and the power factors.

This emphasizes the advantage of compensating for the phase difference of the condensers, for when the galvanometer current is zero it is certain that the balance depends only on the constants of the four arms of the bridge, and that no external disturbance is appreciable.

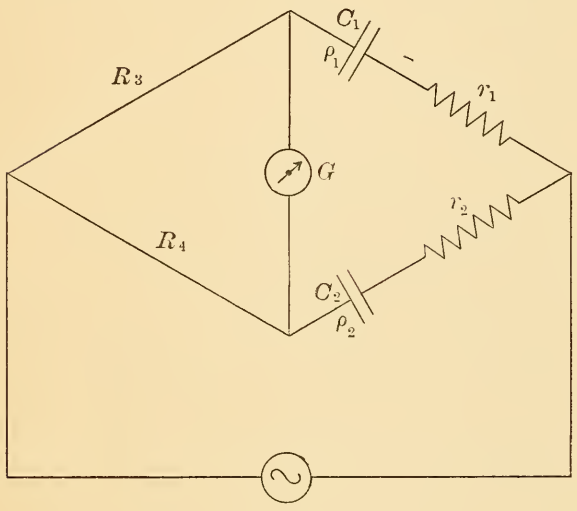

Fig. 8.

Wien derives the conditions for balance in this case on the assumption that the absorption may be represented by a resistance in *parallel* with the condenser. I have, however, adopted the convention of a *series* resistance, because it leads to simpler and more symmetrical formulæ.

To avoid ascertaining which of the two condensers has the smaller power factor, it is convenient to insert resistance in series with each condenser, as shown in Fig. 8.

Derivation of the formulæ.—Let the absorptions of the condensers be represented by fictitious resistances ρ_1 and ρ_2. Then the impedances of the different arms are

$$a_1 = (\rho_1 + r_1) - \frac{i}{pC_1}$$

$$a_2 = (\rho_2 + r_2) - \frac{i}{pC_2}$$

$$a_3 = R_3$$

$$a_4 = R_4 \text{ where } i = \sqrt{-1}$$

It has been shown by Lord Rayleigh[9] and others that the condition for balance in such a bridge is

$$a_2a_3 - a_1a_4 = 0 \tag{4}$$

We have, therefore,

$$R_3\left[(\rho_2+r_2)-\frac{i}{pC_2}\right]-R_4\left[(\rho_1+r_1)-\frac{i}{pC_1}\right]=0$$

Separating the real and imaginary parts,

$$R_3(\rho_2+r_2)-R_4(\rho_1+r_1)=0$$

$$-\frac{R_3}{pC_2}+\frac{R_4}{pC_1}=0$$

$$\therefore \frac{\rho_2+r_2}{\rho_1+r_1}=\frac{R_3}{R_4} \tag{5}$$

$$\frac{C_2}{C_1}=\frac{R_3}{R_4} \tag{6}$$

From these it follows that

$$\frac{\rho_2+r_2}{\rho_1+r_1}=\frac{C_1}{C_2}$$

$$\therefore pC_2(\rho_2+r_2)=pC_1(\rho_1+r_1)$$

or

$$pC_2\rho_2-pC_1\rho_1=pC_1r_1-pC_2r_2$$

which by (1) becomes

$$\tan\theta_2-\tan\theta_1=pC_1r_1-pC_2r_2$$

For such small angles, however, we may write

$$\tan\theta_2-\tan\theta_1=\tan(\theta_2-\theta_1)$$

and the above formula becomes

$$\tan(\theta_2-\theta_1)=pC_1r_1-pC_2r_2 \tag{7}$$

[9] Proc. Roy. Soc. **49**, p. 203; 1885.

To test the approximation, let us take the unfavorable case

$$\theta_1 = 0^\circ\ 3' \qquad \theta_2 = 1^\circ\ 0'$$

Then $$\tan\theta_1 \tan\theta_2 = 0.000015$$

This quantity is negligible in the measurement of power factor.

It may easily be shown that (7) is equivalent to Wien's expression (*B*) for the power factor. For that case $\theta_2 = 0$, and therefore $\rho_2 = 0$, so that (7) becomes

$$pC_2r_2 = \tan\theta_1$$

Wien's expression is

$$p^2C_1C_2W_1r_2 = 1$$

which may be written

$$pC_2r_2 = \frac{1}{pC_1W_1}$$

and by (2) we have

$$pC_2r_2 = \tan\theta_1$$

Sources of Error.

(*a*) Inductance or capacity of R_3 and R_4.

(*b*) Error in the ratio $R_3 : R_4$.

(*c*) Inductance or capacity of r_1 and r_2.

(*d*) Electrostatic induction between the bridge and its surroundings.

We will consider these separately, since their elimination is accomplished in different ways.

(*a*) **Inductance or capacity of R_3 and R_4.**—For this case

$$a_3 = R_3 + ipl_3$$
$$a_4 = R_4 + ipl_4$$

Then for balance of the bridge

$$(R_3 + ipl_3)\left[(\rho_2 + r_2) - \frac{i}{pC_2}\right] - (R_4 + ipl_4)\left[(\rho_1 + r_1) - \frac{i}{pC_1}\right] = 0.$$

The real part of this gives

$$R_3(\rho_2 + r_2) + \frac{pl_3}{pC_2} - R_4(\rho_1 + r_1) - \frac{pl_4}{pC_1} = 0$$

which from (6) and (1) may be written approximately

$$\tan(\theta_2-\theta_1)=(pC_1r_1-pC_2r_2)+\left(\frac{pl_4}{R_4}-\frac{pl_3}{R_3}\right) \tag{8}$$

$$=(pC_1r_1-pC_2r_2)+(\tan\phi_4-\tan\phi_3) \tag{8a}$$

The imaginary part gives

$$-\frac{R_3}{pC_2}+pl_3(\rho_2+r_2)+\frac{R_4}{pC_1}-pl_4(\rho_1+r_1)=0$$

or

$$\frac{C_2}{C_1}=\frac{R_3}{R_4}+\frac{p^2C_2}{R_4}\left\{l_4(\rho_1+r_1)-l_3(\rho_2+r_2)\right\}$$

which may be written approximately

$$\frac{C_2}{C_1}=\frac{R_3}{R_4}\left[1+\frac{pl_4}{R_4}\cdot pC_1(\rho_1+r_1)-\frac{pl_3}{R_3}\cdot pC_2(\rho_2+r_2)\right] \tag{9}$$

$$=\frac{R_3}{R_4}\left[1-\tan\phi_4\tan(90^\circ-\phi_1)+\tan\phi_3\tan(90^\circ-\phi_2)\right] \tag{9a}$$

where ϕ_1, ϕ_2, etc., are the phase differences of the currents and impressed electromotive forces of the arms 1, 2, etc., an angle of *lag* being taken as positive.

As an example, suppose

$C_2=C_1=1$ mf	$r_1=1$ ohm
$R_3=R_4=1{,}000$ ohms	$p=628$
$\rho_1=49$ ohms	$l_3=-0.00004$ henry
$\rho_2=5$ ohms	$l_4=-0.0015$ henry

A *negative* inductance l signifies that the capacity effect in the coil is predominant, and that its effect on the phase of the current is equal and opposite to that of an inductance l.

This case is realized where R_4 consists of a single coil of bifilar winding, and R_3 is made up of a number of coils, the subdivision reducing its effective electrostatic capacity.

From equation (5) r_2 = approximately 45 ohms, and by (9) the correction to the capacity ratio is nearly 3 parts in 100,000.

By (8), $\tan(\theta_2-\theta_1)=0.0276-.00092$, that is, the correction to the difference of phase of the two condensers is more than 3 per cent, a quantity which would be important if the power factors of the two condensers were nearly equal. However, by taking for R_3 and R_4 coils wound similarly, and by making another setting with R_3 and R_4 interchanged, this error may be much reduced.

(*b*) **Error in the ratio R_3: R_4.**—Suppose that R_3 and R_4 are each divided into two parts such that

$$R_3=W_3+w_3$$
$$R_4=W_4+w_4$$

where W_3 and W_4 have the same nominal value, and are nearly equal to R_3 and R_4, and where w_3 and w_4 are small variable resistances of perhaps 100 ohms. Then if the latter be fairly well adjusted, W_3 and W_4 do not need to be accurately known, provided that the bridge be balanced a second time with W_3 and W_4 interchanged, but with w_3 and w_4 in their original positions.

For, if $W_3=W+\delta_3$ and $W_4=W+\delta_4$, where δ_3 and δ_4 are the differences between W_3 and W_4 and their nominal value W, then

$$\frac{C_1}{C_2}=\frac{W_4+w_4'}{W_3+w_3'}=\frac{W_3+w_4''}{W_4+w_3''}$$

or

$$\frac{C_1}{C_2}=\frac{W+\delta_4+w_4'}{W+\delta_3+w_3'}=\frac{W+\delta_3+w_4''}{W+\delta_4+w_3''}$$

By addition we have

$$\frac{C_1}{C_2}=\frac{2W+(w_4'+w_4'')+(\delta_3+\delta_4)}{2W+(w_3'+w_3'')+(\delta_3+\delta_4)}$$

$$=\frac{W+\frac{1}{2}(w_4'+w_4'')+\frac{1}{2}(\delta_3+\delta_4)}{W+\frac{1}{2}(w_3'+w_3'')+\frac{1}{2}(\delta_3+\delta_4)} \qquad (10)$$

or, very nearly

$$\frac{C_1}{C_2}=\frac{W+\frac{1}{2}(w_4'+w_4'')}{W+\frac{1}{2}(w_3'+w_3'')} \qquad (10a)$$

To calculate the ratio of the capacities from (10*a*) it is necessary to know only the means of the settings of w_3 and w_4 and the nominal

value W of the main coils. Usually only one of the resistances w_3 and w_4 would be varied and the formula would be somewhat simplified.

As a test of error from using (10*a*) let us take as a rather extreme case

$$\delta_3 = 1 \text{ ohm} \qquad W + \frac{w_4' + w_4''}{2} = 1100 \text{ ohms}$$

$$\delta_4 = 2 \text{ ohms} \qquad W + \frac{w_3' + w_3''}{2} = 1050 \text{ ohms.}$$

From formula (10) $\frac{C_1}{C_2} = 1.04765$.

From formula (10*a*) $\frac{C_1}{C_2} = 1.04755$, an error of only 1 in 10,000. With well-adjusted coils and with errors δ_3 and δ_4 of opposite sign the error would be much smaller.

(*c*) **Inductance or capacity of r_1 and r_2.**—Formula (7) shows that for a given difference of phase $(\theta_2 - \theta_1)$, the difference of the resistances r_1 and r_2 will be greater the smaller the capacities measured. For instance, for

$$C_1 = C_2 = 0.001 \text{ mf}$$
$$p = 628$$
$$\theta_2 - \theta_1 = 0^\circ 5'$$

we will have $(r_1 - r_2) = 2{,}316$ ohms. Examples will be given below to show that much greater values of θ may occur in small mica condensers. Measurements of the change of phase produced by coils of 10,000 ohms and greater show they have quite an appreciable capacity. For instance, the equivalent capacity of a 10,000-ohm coil—that is, the capacity which must be shunted across the terminals of a resistance of 10,000 ohms in order to produce a phase displacement equal to that observed in the coil—was found to be 0.0019 mf. For a 100,000-ohm coil the value was 0.00058 mf. It is therefore interesting to investigate the effect of introducing such capacities into the arm with the condenser.

The impedance of a resistance r in parallel with a capacity k is

$$\frac{-\frac{i}{pk}r}{r - \frac{i}{pk}} = -\frac{ir}{pkr - i} = \frac{r - ipkr^2}{1 + p^2k^2r^2}$$

Therefore, putting

$$a_1=\left(\rho_1-\frac{i}{pC_1}\right)+\frac{r_1-ipk_1r_1^2}{1+p^2k_1^2r_1^2}$$

$$a_2=\left(\rho_2-\frac{i}{pC_2}\right)+\frac{r_2-ipk_2r_2^2}{1+p_2k_2^2r_2^2}$$

$$a_3=R_3$$

$$a_4=R_4$$

$$R_3\left[\left(\rho_2+\frac{r_2}{1+p^2k_2^2r_2^2}\right)-i\left(\frac{1}{pC_2}+\frac{pk_2r_2^2}{1+p^2k_2^2r_2^2}\right)\right]$$

$$-R_4\left[\left(\rho_1+\frac{r_1}{1+p^2k_1^2r_1^2}\right)-i\left(\frac{1}{pC_1}+\frac{pk_1r_1^2}{1+p^2k_1^2r_1^2}\right)\right]=0.$$

Separating the real and imaginary parts, we have

$$R_3\left(\rho_2+\frac{r_2}{1+p^2k_2^2r_2^2}\right)-R_4\left(\rho_1+\frac{r_1}{1+p^2k_1^2r_1^2}\right)=0 \qquad (11)$$

$$-R_3\left(\frac{1}{pC_2}+\frac{pk_2r_2^2}{1+p^2k_2^2r_2^2}\right)+R_4\left(\frac{1}{pC_1}+\frac{pk_1r_1^2}{1+p^2k_1^2r_1^2}\right)=0 \qquad (12)$$

Equation (12) may be written

$$\frac{C_2}{C_1}=\frac{R_3}{R_4}\left[1+\frac{p^2C_2k_2r_2^2}{1+p^2k_2^2r_2^2}-\frac{p^2C_1k_1r_1^2}{1+p^2k_1^2r_1^2}\right]$$

or, neglecting the quantities $p^2k_1^2r_1^2$ and $p^2k_2^2r_2^2$ with respect to unity,

$$\frac{C_2}{C_1}=\frac{R_3}{R_4}\left[1+(p^2C_2k_2r_2^2-p^2C_1k_1r_1^2)\right] \qquad (13)$$

and from (11), using the approximation $\frac{R_3}{R_4}=\frac{C_2}{C_1}$ we may derive

$$\tan(\theta_2-\theta_1)=\frac{pC_1r_1}{1+p^2k_1^2r_1^2}-\frac{p_2C_2r_2}{1+p^2k_2^2r_2^2}$$

or, approximately,

$$\tan(\theta_2-\theta_1)=pC_1r_1[1-(pk_1r_1)^2]-pC_2r_2[1-(pk_2r_2)^2] \qquad (14)$$

For large capacities, where r_1, r_2, and, therefore, k_1, k_2, are small, the correction terms in (13) and (14) will be negligible. In any case, if the condensers be of nearly equal capacity and power factor, the corrections will be unimportant, since r_1 and r_2 and, therefore, k_1 and k_2 will be nearly equal.

When, however, condensers with capacities as small as 0.001 mf, and with quite different power factors, are to be compared, these corrections may become appreciable.

In one extreme case, when a poor mica condenser of 0.003 mf was being compared with a good mica condenser of the same capacity, the uncorrected capacity was found (by calculation from the approximate values of the capacities of r_1 and r_2) to be in error by 7 per cent. The correction to the measured value of the power factor, due to the same cause, was 15 per cent. Usually, however, the errors are much smaller.

(*d*) **Electrostatic capacity of the bridge.**—The errors, due to the electrostatic capacity of the different parts of the bridge with respect to the earth, are in some cases troublesome, but may be largely eliminated. Owing to this cause the balance point of the bridge depends on the potential of the points A and B, Fig. 4, with respect to the earth, and changes when resistance is placed unsymmetrically in the galvanometer circuit.

Although these effects are especially important only when capacities smaller than 0.1 mf are being compared, the following method for the elimination of such errors is so simple that it is recommended in all cases.

The substitution method.—To compare two nearly equal condensers, an auxiliary condenser C_2 is employed. The condenser to be tested, C_1', and the standard, C_1'', are then successively balanced against C_2.

Neglecting residual inductances or capacities in the resistances, we will have from (6)

$$\frac{C_1'}{C_2}=\frac{R_4}{R_3'} \text{ and } \frac{C_1''}{C_2}=\frac{R_4}{R_3''}$$

Therefore

$$\frac{C_1'}{C_1''}=\frac{R_3''}{R_3'} \qquad (15)$$

where R_3' and R_3'' are the two settings of R_3 found in the two measurements.

For the power factor we have from (7)

$$\begin{aligned} \tan\theta_1' - \tan\theta_2 &= pC_2r_2 - pC_1'r_1' \\ \tan\theta_1'' - \tan\theta_2 &= pC_2r_2 - pC_1''r_1'' \\ \therefore \tan\theta_1' - \tan\theta_1'' &= pC_1''r_1'' - pC_1'r_1' \end{aligned}$$

or

$$\tan(\theta_1' - \theta_1'') = pC_1''r_1'' - pC_1'r_1' \qquad (16)$$

$$= pC_1''\left(r_1'' - \frac{R_3''}{R_3'}\cdot r_1'\right) \qquad (16a)$$

r_1' and r_1'' being the two observed values of r_1.

Either (16) or (16a) is convenient to use, the logarithms of the capacities C_1', C_1'', and of their ratio $\frac{R_3''}{R_3'}$ having been already found in the calculation of the capacities. From its symmetry, formula (16) is perhaps to be preferred, when a number of condensers have been measured, the quantity pC_1r_1 being calculated for each of the condensers, and subtracted from the corresponding quantity for the standard.

Each of the ratio arms consists of two parts—a resistance W whose nominal value is the same for both arms, and a much smaller variable resistance w, one of which, w_3, is used to make the final setting. With condensers whose capacities do not differ by more than a few per cent, the resistance needs only to be approximately known, since the difference of the capacities is proportional to the difference in the observed values of w_3. Additional measurements may be taken with the two coils W reversed (independently of w_3 and w_4) as previously described, but this is now of advantage only to obtain check measurements for reducing the effect of the accidental errors in balancing the bridge. The same end is reached by taking an additional pair of readings with the condenser arms interchanged.

Of the series resistances r_1, r_2, the latter is kept constant and the phases of the condenser currents are brought into unison by adjusting r_1. The difference in the power factors of the two condensers is, therefore, measured by the difference of the settings of r_1, and, consequently, the leads in the condenser arms do not need to be known.

Of course, if any condenser is used with permanent leads of its own, their resistance should be determined and a correction applied to the apparent power factor if necessary.

The effects of the electrostatic capacity of the bridge itself will enter into the measurements on the two condensers in the same manner, since they are both measured in the same arm of the bridge; and, if their capacities are not very different, the value of the error in the two cases will be sensibly the same and will not affect either the measurement of the capacity or the power factor. If, however, the condensers are of small and quite unequal capacity, the electrostatic effects may be different enough as a result of the necessary change in R_3, when one condenser is substituted for the other, to cause an appreciable error. A modification of the method which overcomes this difficulty will be described later.

In deriving the formulæ for the substitution method, the residual inductances and capacities of the ratio arms have been neglected. This is entirely allowable, since the resistances included in R_3' and R_3'' are identical except for the small resistance $(R_3' - R_3'')$ which has negligible inductance. The correction terms, therefore, in (8) and (9) do not appreciably change when one condenser is substituted for the other, and since their absolute value has been made small, by using similarly wound coils in R_3 and R_4, this error may be considered as entirely eliminated.

In order to avoid introducing capacity into the condenser arms outside of the condensers themselves, Professor Rosa has suggested the following method, which may be regarded as more generally applicable than the Series Resistance Method. It presents, however, some practical inconveniences when condensers of very large power factor are to be measured.

4. SERIES INDUCTANCE METHOD.

As its name suggests, this method makes use of a variable inductance to compensate for the difference of phase of the currents in the condensers. It has the advantage that the compensating device is not placed in the arm with the condenser, and can not, therefore, modify the capacity of that branch. It is, for this reason, of especial value in the comparison of very small capacities.

Fig. 9 shows the arrangement of the bridge when two inductances are employed, one of fixed value and the other variable. Only one inductance is necessary, provided the condensers are not of too nearly equal power factor. In such a case, the single inductance, which must of course be variable, is placed in the ratio arm which is adjacent to the condenser with the larger power factor. By using *two* coils, however, the necessity for knowing, in advance, which condenser has the greater absorption is removed. Further, when the condensers are of nearly equal power factor, and, consequently, the

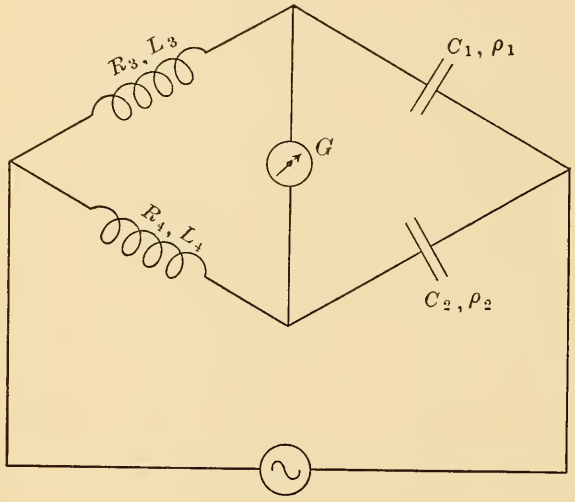

Fig. 9.

value of the inductance of the single coil would be small, the insertion of a coil in the other arm obviates the inconvenience of working with small inductances, and the inductance of the leads does not need to be considered.

Derivation of the formulæ.—To derive the conditions for a balance of the bridge, we proceed as before. From Fig. 9

$$a_1=\rho_1-\frac{i}{pC_1} \qquad a_3=R_3+ipL_3$$

$$a_2=\rho_2-\frac{i}{pC_2} \qquad a_4=R_4+ipL_4$$

and substituting in the equation $a_1a_4-a_2a_3=0$, we have

$$(R_3+ipL_3)\left(\rho_2-\frac{i}{pC_2}\right)-(R_4+ipL_4)\left(\rho_1-\frac{i}{pC_1}\right)=0$$

Separating the real and imaginary parts, we obtain

$$R_3\rho_2 - R_4\rho_1 + \frac{L_3}{C_2} - \frac{L_4}{C_1} = 0 \tag{17}$$

$$-\frac{R_3}{pC_2} + \frac{R_4}{pC_1} + pL_3\rho_2 - pL_4\rho_1 = 0 \tag{18}$$

From (17) we have

$$\tan(\theta_1 - \theta_2) = \frac{pL_3}{R_3} - \frac{pL_4}{R_4} \tag{19}$$

and from (18)

$$\frac{C_1}{C_2} = \frac{R_4}{R_3} + \frac{p^2 C_1}{R_3}(L_3\rho_2 - L_4\rho_1)$$

$$= \frac{R_4}{R_3} + \frac{pL_3}{R_3} pC_2\rho_2 \frac{C_1}{C_2} - \frac{pL_4}{R_4} pC_1\rho_1 \frac{R_4}{R_3}$$

$$= \frac{R_4}{R_3}\left[1 + \frac{pL_3}{R_3}\tan\theta_2 - \frac{pL_4}{R_4}\tan\theta_1\right] \tag{20}$$

$$= \frac{R_4}{R_3}\left[1 + \tan\phi_3 \tan\theta_2 - \tan\phi_4 \tan\theta_1\right] \tag{20a}$$

Although the usual simple formula (3) for the ratio of the capacities does not hold exactly, the correction terms in (20) are exceedingly small in practice. As an example let us take

$R_3 = R_4 = 1{,}000$ ohms
$L_4 = 0.01$ henry
$p = 628$
$\theta_1 = 0^\circ.5'$ and $\therefore \tan\theta_1 = 0.001454$
$\theta_2 = 0^\circ\ 1'$ $\tan\theta_2 = 0.000291$

We then find

$$\frac{pL_4}{R_4} = 0.00628, \text{ and from (19)}$$

$$\frac{pL_3}{R_3} = \tan 4' + 0.00628 = 0.001792$$

The correction term in (20) is, therefore,

$$0.001792 \times 0.000291 - 0.00628 \times 0.001454 = -0.0000085$$

which is negligible.

What has been said concerning the elimination of errors in the Series Resistance Method is applicable here. As in the previous method, also, the comparison of the condenser with the standard by means of a substitution method is to be recommended.

The substitution method.—Using the previous nomenclature, we may derive from (19)

$$\tan(\theta_1' - \theta_2) = p\frac{L_3'}{R_3'} - \frac{p\,L_4}{R_4}$$

$$\tan(\theta_1'' - \theta_2) = \frac{pL_3''}{R_3''} - \frac{pL_4}{R_4}$$

$$\text{and } \therefore \tan(\theta_1' - \theta_1'') = \frac{pL_3'}{R_3'} - \frac{pL_3''}{R_3''} \qquad (21)$$

and from (20)

$$\frac{C_1'}{C_2} = \frac{R_4}{R_3'}\left[1 + \tan\theta_2 \cdot \frac{pL_3'}{R_3'} - \tan\theta_1' \cdot \frac{pL_4}{R_4}\right]$$

$$\frac{C_1''}{C_2} = \frac{R_4}{R_3''}\left[1 + \tan\theta_2 \cdot \frac{pL_3''}{R_3''} - \tan\theta_1'' \cdot \frac{pL_4}{R_4}\right]$$

or, approximately,

$$\frac{C_1'}{C_1''} = \frac{R_3''}{R_3'}\left[1 + \tan\theta_2\left(\frac{pL_3'}{R_3'} - \frac{pL_3''}{R_3''}\right) - \frac{pL_4}{R_4}\tan(\theta_1' - \theta_1'')\right]$$

which by (21) becomes

$$\frac{C_1'}{C_1''} = \frac{R_3''}{R_3'}\left[1 + \tan(\theta_1' - \theta_1'')\left\{\tan\theta_2 - \frac{pL_4}{R_4}\right\}\right] \qquad (22)$$

The correction factor in this equation is, as before, very small, so that usually we calculate the capacity ratio by the simple formula (3). It is to be noticed that the correction depends almost entirely on the power factors of the two condensers, and their difference. As an example let us take the constants of the preceding problem. The correction term in (22) becomes

$$\tan 4'\,(\tan 1' - 0.00628) = -0.000007$$

which is usually negligible.

The substitution method just described possesses the following advantages:

(*a*) Ordinary errors of adjustment of the resistances R_3 and R_4 are of negligible effect.

(*b*) The resistances of the inductances need not be accurately measured. This is a great advantage, since inductances are generally wound with copper, and it is difficult to determine accurately enough the temperature of the coils. This advantage is not shared by the method where the condensers are compared directly.

(*c*) Errors due to the electrostatic capacity of the bridge (see p. 389) are practically eliminated.

(*d*) The absolute values of the inductances need not be known very accurately. It is merely necessary that the relative values of the inductance in L_3 be determined. If L_3 is a well-calibrated variable inductance this condition can be fulfilled. When, however, the power factor of the condenser to be tested differs very much from that of the standard, some inconvenience may be experienced in finding the proper value of the inductance. This difficulty may be lessened, when a limited number of fixed inductances are at hand, by varying both ratio arms by equal amounts to aid in obtaining a balance. If, however, the difference in θ_1' and θ_1'' is much over a degree of arc, the frequency is below 100 cycles per second, and the condensers are not smaller than about 0.01 mf, the Series Resistance Method will be found more convenient.

The formula (21) for the power factor is easy to use. Having calculated the capacities, it is only necessary to determine the quantity $\frac{pL_3}{R_3}$ for each condenser and to take the differences. With this method it is easier to judge of the order of magnitude of the power factor, while the observations are being taken, than with the Series Resistance Method, because the setting of the inductance does not depend on the *magnitude* of the capacity.

5. PARALLEL RESISTANCE METHOD.

In this method the difference of phase in the condenser currents is compensated by a variable resistance R in parallel with the condenser which has the smaller power factor. From equation (2) it follows that, at a frequency of 100 cycles, to represent the absorption of

a 1-microfarad condenser, for which $\theta = 0° 5'$, the fictitious resistance W must be taken as greater than 1 megohm. The smaller the capacity and the power factor the greater will this resistance be. In order, therefore, to avoid using extremely large resistances, it is better to place 100,000 ohms in parallel with the condenser to be tested, in which case the adjustable resistance in parallel with the standard will be of convenient value. This arrangement of the bridge is shown in Fig. 10, the absorption resistances W_1 and W_2 of the condensers being shown dotted.

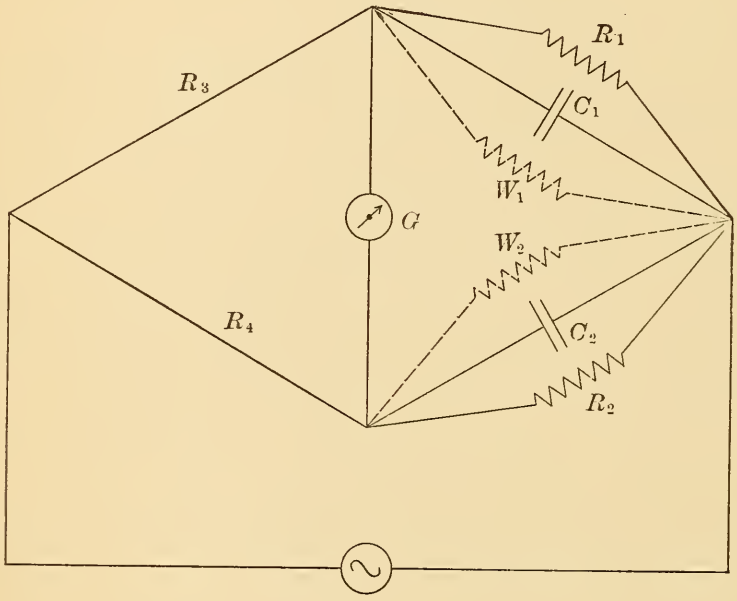

Fig. 10.

This is similar to a bridge method given by Wien for the simultaneous measurement of two capacities. For this purpose, however, R_1 and R_2 must be taken so small that the effect of W_1 and W_2 on the resistances of those arms is small or negligible.

Derivation of the formulæ.—In Fig. 10 let N_1 and N_2 be the joint resistances of the condenser arms.

$$\frac{1}{N_1} = \frac{1}{W_1} + \frac{1}{R_1}$$

$$\frac{1}{N_2} = \frac{1}{W_2} + \frac{1}{R_2} \qquad (23)$$

We have, then,

$$a_1 = \frac{1}{\frac{1}{N_1} + \frac{1}{\frac{i}{-pC_1}}} = \frac{1}{\frac{1}{N_1} + ipC_1}$$

$$a_2 = \frac{1}{\frac{1}{N_2} + ipC_2}$$

$$a_3 = R_3$$
$$a_4 = R_4$$

For balance of the bridge

$$\frac{R_3}{\frac{1}{N_2} + ipC_2} - \frac{R_4}{\frac{1}{N_1} + ipC_1} = 0$$

or

$$\frac{R_3}{N_1} + ipC_1R_3 - \frac{R_4}{N_2} - ipC_2R_4 = 0$$

Separating the real and imaginary parts

$$\frac{R_3}{N_1} = \frac{R_4}{N_2} \tag{24}$$

$$\frac{C_1}{C_2} = \frac{R_4}{R_3} \tag{25}$$

From these it follows that

$$\frac{N_1}{N_2} = \frac{C_2}{C_1} \quad \text{or} \quad \frac{\frac{1}{N_1}}{\frac{1}{N_2}} = \frac{\frac{1}{C_2}}{\frac{1}{C_1}}$$

Substituting for N_1 and N_2 their values in (23) this becomes

$$\frac{\frac{1}{W_1} + \frac{1}{R_1}}{\frac{1}{W_2} + \frac{1}{R_2}} = \frac{\frac{1}{C_2}}{\frac{1}{C_1}}$$

$$\frac{1}{pC_1W_1} - \frac{1}{pC_2W_2} = \frac{1}{pC_2R_2} - \frac{1}{pC_1R_1}$$

or

$$\tan\theta_1 - \tan\theta_2 = \frac{1}{pC_2R_2} - \frac{1}{pC_1R_1}$$

and very approximately

$$\tan(\theta_1-\theta_2)=\frac{1}{pC_2R_2}-\frac{1}{pC_1R_1} \tag{26}$$

In order that we may use the simple formula (25) it is necessary that the capacities k_1, k_2 of the resistances R_1, R_2 shall be negligible compared with C_1 and C_2. If this condition is not fulfilled, these capacities, which are in parallel with the condenser arms, must be included in (25), which becomes

$$\frac{C_1+k_1}{C_2+k_2}=\frac{R_4}{R_3}$$

or very approximately

$$\frac{C_1}{C_2}=\frac{R_4}{R_3}\left[1+\left(\frac{k_2}{C_2}-\frac{k_1}{C_1}\right)\right] \tag{27}$$

If the power factors are nearly equal, R_1 and R_2 will be also nearly equal, and by using similarly wound coils in these resistances, the differences between k_1 and k_2 will be small, so that the correction term in (27) may become negligible.

The substitution method.—This is the same in principle as in the Series Resistance and Series Inductance Methods. Adopting the usual nomenclature, the formulæ for this case follow at once from (26) and (27). They are

$$\frac{C_1'}{C_1''}=\frac{R_3''}{R_3'}\left[1+\left(\frac{k_1''}{C_1''}-\frac{k_1'}{C_1'}\right)\right] \tag{28}$$

$$\tan(\theta_1'-\theta_1'')=\frac{1}{pC_1''R_1''}-\frac{1}{pC_1'R_1'} \tag{29}$$

The substitution method possesses the following previously discussed advantages:

(*a*) The ratio arms need not be accurately known.

(*b*) The effect of residual inductances or capacities in R_3 and R_4 is eliminated.

(*c*) Electrostatic induction effects are eliminated.

And, further, (*d*), by using this arrangement, the correction term of (28) is much reduced, and in many cases becomes negligible, since k_1' and k_1'' differ only by the value of the capacity of the resistance $(R_1'-R_1'')$.

6. ADAPTATION OF ANDERSON'S METHOD.

In each of the three methods just described a simple bridge is used, the condensers being included in two of the arms, and the ratio coils in the other two, and their differences lie in the means by which the difference of phase of the condenser currents is brought to zero. The scheme about to be described is considerably different in principle, and is, therefore, aside from its other advantages, especially valuable as a check on the other three methods.

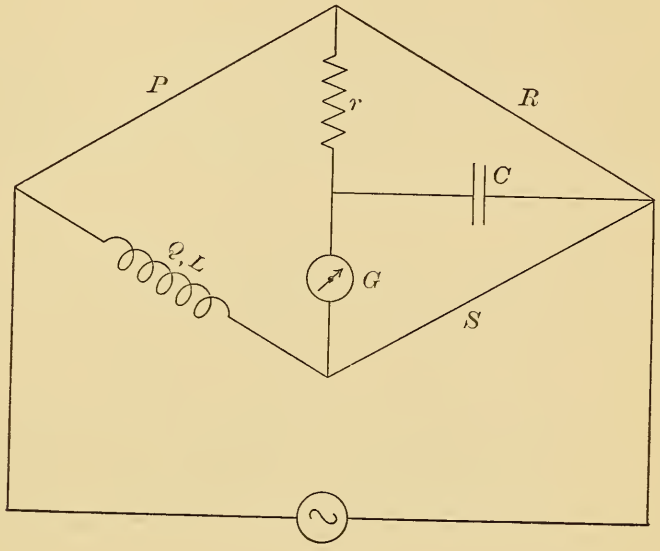

Fig. 11.

The conditions for balance in the ideal Anderson's bridge, Fig. 11, are[10]

$$L = CS\left[\frac{r(P+R)}{R} + P\right] \tag{30}$$

$$\frac{Q}{S} = \frac{P}{R} \tag{31}$$

The expression for L is very little modified by the power factor of the condenser,[11] but under certain circumstances the deviation of the resistance Q from the value given by (31) may become considerable.

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[10] Anderson, Phil. Mag., **31**, p. 329; 1891.

[11] Rosa and Grover, this Bulletin, **1**, p. 312, equations 18 and 20; 1905.

The modified formulæ are

$$L=CS\left[\frac{r(P+R)}{R}+P\right](1-\tan^2\theta) \tag{32}$$

$$Q=\frac{PS}{R}+p^2L\rho C=\frac{PS}{R}+pL\tan\theta \tag{33}$$

or, when $P=R$,

$$L=CS\,(2r+P)\,(1-\tan^2\theta) \tag{34}$$

$$Q=S+pL\tan\theta \tag{35}$$

where θ is the phase difference of the condenser used in the bridge.

If the bridge be balanced successively with two condensers having a phase difference $(\theta_1-\theta_2)$, the same inductance being used in both cases, we obtain at once the ratio of the capacities and the difference of their power factors.

For, from (34)

$$L=C_1S\,(2r_1+P)\,(1-\tan^2\theta_1)$$

$$L=C_2S\,(2r_2+P)\,(1-\tan^2\theta_2)$$

$$\therefore\ \frac{C_2}{C_1}=\frac{2r_1+P}{2r_2+P}\cdot\frac{(1-\tan^2\theta_1)}{(1-\tan^2\theta_2)}$$

or, very approximately,

$$\frac{C_2}{C_1}=\frac{2r_1+P}{2r_2+P}\left[1+(\tan^2\theta_2-\tan^2\theta_1)\right] \tag{36}$$

and in many cases it is sufficient to write

$$\frac{C_2}{C_1}=\frac{2r_1+P}{2r_2+P} \tag{36a}$$

From (35)

$$Q_1=S+pL\tan\theta_1$$

$$Q_2=S+pL\tan\theta_2$$

$$\therefore\ Q_2-Q_1=\Delta Q=pL\,(\tan\theta_2-\tan\theta_1)$$

or nearly enough

$$\Delta Q=pL\tan(\theta_2-\theta_1) \tag{37}$$

from which is obtained

$$\tan(\theta_2-\theta_1)=\frac{\Delta Q}{pL}=\frac{\Delta Q'}{pL} \qquad (38)$$

where $\Delta Q'$ is the observed change in the resistance Q' included in the Q arm with the coil.

It is very seldom that the correction factor in (36) need be applied, since for the rare case that $(\theta_2-\theta_1)=1°$ its value is only 3 parts in 10,000, and this decreases nearly with the square of $(\theta_2-\theta_1)$.

This is inherently a substitution method, and possesses the advantages that, for condensers of the same nominal value—

(*a*) Only the relative values of r_1 and r_2 need be *closely* known in the determination of the ratio of the capacities. It is sufficient, with fairly well adjusted resistances, to use the nominal values of P and r.

(*b*) Since only the small change in Q is desired, this is given accurately enough by the difference of the nominal values of the two observed values of Q', the resistance exclusive of the resistance of the coil.

(*c*) Errors due to the change of resistance of the inductance coil with the frequency are eliminated, together with the effect of residual inductances or capacities in the bridge arms.

(*d*) Errors due to the electrostatic capacity of the bridge are practically eliminated.

Formula (38) shows that in order to obtain $(\theta_2-\theta_1)$ accurately, the change ΔQ in the resistance of the Q arm, when one condenser is substituted for the other, must be large enough to be determined with precision.

For a given capacity at a given frequency, this requires that L be chosen large enough to satisfy this condition. The higher the frequency the smaller the value of the inductance necessary to obtain a given value of ΔQ. In addition to these considerations it must be remembered that the arrangement of the bridge is not sensitive when large inductances are used with small condensers, and that the sensitiveness of the balance of the Q arm depends on the relative values of Q and pL. With such a large number of variables it is only a matter of trial to obtain a satisfactory arrangement for the case in hand.

Some trouble is met when the temperature of the copper wire of the inductance is changing, especially if the coil be of fairly high resistance. If, however, several measurements be made on the two condensers alternately, this error is eliminated.

7. THE DETERMINATION OF THE ABSOLUTE VALUE OF THE POWER FACTOR.

If a condenser is at hand whose power factor is known at a number of frequencies and temperatures, so that the value of θ corresponding to any particular working conditions can be obtained, then it is easy to determine θ for any other condenser of the same nominal value. By one of the four methods just described, the difference $(\theta_1' - \theta_1'')$ is determined. Knowing, then, the resistance of the leads of the condenser which is being tested, the observed value can be corrected, and the value $\theta_1 - \theta_1''$, corresponding to the condenser alone, is obtained. This quantity is the phase difference between the condenser and the standard plus its leads. From the known value, θ_0, of the angle of the standard, and from the resistance of its leads, the value θ_1'', corresponding to the standard plus its leads, is obtained, and combining with this the value of $(\theta_1 - \theta_1'')$ we obtain the angle θ_1 for the condenser in question. The correction ψ for a lead resistance, a, is calculated by the relation

$$\tan \psi = p C a$$

For condensers of large capacity at high frequency a resistance of only 0.001 ohm in the leads is appreciable. For a condenser of 1 mf capacity at 1,000 cycles the correction for a lead resistance of 0.001 ohm is 1″.3. This illustrates how important it is to know the resistance of the leads.

To avoid changes in the apparent power factor of a condenser, plug contacts on the top of its containing case must be kept in good condition, and to obtain a correct value of the power factor of the condenser itself the lead wires from the condenser plates to the terminal blocks or binding posts should be of known or negligible resistance.

Power factor measured by stepping up from an air condenser.—A well-insulated air condenser, the resistance of whose leads is too

small to appreciably affect the phase, may be considered as having a zero power factor.[12] The power factor due to an insulation resistance R may be calculated by the formula $\tan \theta = \frac{1}{pCR}$ in order to determine, in any case, whether the effect of leakage is negligible.

Such an air condenser may evidently be employed to determine the absolute value of the power factor of a condenser of the same nominal value. Air condensers, however, become bulky when their capacity exceeds 0.02 or 0.03 mf. The Bureau of Standards possesses two air condensers of 0.02 mf each, and two of 0.03 mf each, which, when joined in parallel, allow the power factors of condensers as large as 0.1 mf to be directly determined. To obtain the power factors of condensers of larger capacity, to be used as standards in condenser comparisons, a stepping-up process has been used. Knowing the value of θ for two or more condensers, we may calculate their joint power factor when connected in parallel.

Let the capacities be C_1, C_2, C_3, etc., and their known phase differences θ_1, θ_2, θ_3, etc.

Then

$$\tan \theta_1 = pC_1r_1$$
$$\tan \theta_2 = pC_2r_2$$
$$\tan \theta_3 = pC_3r_3, \text{ etc.}$$

and the impedances Z_1, Z_2, Z_3, etc., are

$$Z_1 = r_1 - \frac{i}{pC_1} = r_1 + a_1 i \qquad \tan \theta_1 = \frac{r_1}{a_1}$$
$$Z_2 = r_2 - \frac{i}{pC_2} = r_2 + a_2 i \qquad \tan \theta_2 = \frac{r_2}{a_2}$$
$$Z_3 = r_3 - \frac{i}{pC_3} = r_3 + a_3 i \qquad \tan \theta_3 = \frac{r_3}{a_3}$$

Their joint impedance Z is then found from

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots .$$

But

$$\frac{1}{Z_1} = \frac{1}{r_1 + a_1 i} = \frac{r_1 - a_1 i}{r_1^2 + a_1^2} = A_1 - B_1 i$$

[12] At 1,000 cycles a resistance of only 0.01 ohm in the leads shifts the phase of a 0.1-mf condenser by 1″.25.

Similarly

$$\frac{1}{Z_2} = A_2 - B_2 i$$

$$\frac{1}{Z_3} = A_3 - B_3 i, \text{ etc.}$$

$$\therefore \frac{1}{Z} = (A_1 + A_2 + A_3 + \ldots) - i\,(B_1 + B_2 + B_3 + \ldots\ldots)$$

and

$$Z = \frac{(A_1 + A_2 + A_3 + \ldots) + i(B_1 + B_2 + B_3 + \ldots\ldots)}{(A_1 + A_2 + A_3 + \ldots)^2 + (B_1 + B_2 + B_3 + \ldots\ldots)^2}$$

and the joint value θ is given by

$$\tan\theta = \frac{A_1 + A_2 + A_3 + \ldots\ldots}{B_1 + B_2 + B_3 + \ldots\ldots}$$

$$= \frac{\dfrac{r_1}{r_1^2 + \dfrac{1}{p^2 C_1^2}} + \dfrac{r_2}{r_2^2 + \dfrac{1}{p^2 C_2^2}} + \dfrac{r_3}{r_3^2 + \dfrac{1}{p^2 C_3^2}} + \ldots\ldots}{\dfrac{\dfrac{1}{pC_1}}{r_1^2 + \dfrac{1}{p^2 C_1^2}} + \dfrac{\dfrac{1}{pC_2}}{r_2^2 + \dfrac{1}{p^2 C_2^2}} + \dfrac{\dfrac{1}{pC_3}}{r_3^2 + \dfrac{1}{p^2 C_3^2}} + \ldots\ldots}$$

$$= \frac{\dfrac{r_1 p^2 C_1^2}{1 + \tan^2\theta_1} + \dfrac{r_2 p^2 C_2^2}{1 + \tan^2\theta_2} + \dfrac{r_3 p^2 C_3^2}{1 + \tan^2\theta_3} + \ldots\ldots}{\dfrac{pC_1}{1 + \tan^2\theta_1} + \dfrac{pC_2}{1 + \tan^2\theta_2} + \dfrac{pC_3}{1 + \tan^2\theta_3} + \ldots\ldots}$$

Now $(1 + \tan^2\theta_1)$, $(1 + \tan^2\theta_2)$, etc., differ but very little from unity ($= 1.0003$ for $\theta = 1°$) and, further, they occur symmetrically in both numerator and denominator. The error from neglecting these small differences from unity is negligible in measurements of the power factor, and we may write

$$\tan\theta = \frac{p^2 C_1^2 r_1 + p^2 C_2^2 r_2 + p^2 C_3^2 r_3 + \ldots\ldots}{pC_1 + pC_2 + pC_3 + \ldots\ldots}$$

$$= \frac{C_1 \tan\theta_1 + C_2 \tan\theta_2 + C_3 \tan\theta_3 + \ldots\ldots}{C_1 + C_2 + C_3 + \ldots\ldots} \qquad (39)$$

For good mica condensers we may substitute the angles for their tangents, and we have the simple rule, that the *joint angle θ is equal to the weighted mean of the angles θ_1, θ_2, etc.* Thus

$$\theta = \frac{C_1\theta_1 + C_2\theta_2 + C_3\theta_3 + \cdots}{C_1 + C_2 + C_3 + \cdots} \qquad (39a)$$

The joint value for two or more condensers in *series* may be found by a similar procedure, but the formula is not simple, except for the case of n condensers of equal capacity C.

For this case

$$\tan\theta = \frac{\tan\theta_1 + \tan\theta_2 + \tan\theta_3 + \cdots}{n} \qquad (40)$$

If the condensers are not exactly equal, but are of the same nominal value, a rather complicated correction term must be applied to (40). For the special case of two mica condensers of capacities $C(1+a_1)$, $C(1+a_2)$ where C is their nominal value, (40) becomes

$$\tan\theta = \frac{\tan\theta_1 + \tan\theta_2}{2}\left(1 + \frac{a_1 + a_2 + a_1a_2}{2}\right) \qquad (41)$$

In the supposed case a_1 and a_2 will rarely be greater than 1 per cent, and we may write the angles for their tangents. We have, therefore, to take the arithmetical mean of their angles and correct this value by the arithmetical mean of the percentage differences of the capacities from their nominal values, taken with their proper algebraic signs. Since θ may be as small as 1′, the correction will usually be negligible, when

$$\theta = \frac{\theta_1 + \theta_2}{2} \qquad (41a)$$

These formulæ have been applied to several condensers of which the capacities, temperature coefficients, and change of capacity with the frequency had been previously determined. These condensers have been used as standards in testing the methods and in obtaining the power factors and capacities of a large number of other condensers under various conditions. Since, however, so much depended on the determinations of the constants of the standards it was deemed advisable to obtain a check on this stepping-up process. The power factor of a 1 microfarad standard was therefore determined inde-

pendently by means of formula (35), and the value obtained by this method was compared with that derived by stepping up from the 0.1-mf air condenser. As will be seen from Tables IX and X, the agreement was close enough to give confidence in the results of the latter method. Owing to considerations to be presented directly, the results derived from the air condenser will be given the greater weight.

Independent determination of power factor using formula (35).— In an Anderson bridge, in which the noninductive resistances are free from residual inductance or capacity, and in which the inductance and resistance of the coil to be measured do not depend on the frequency, the power factor of the condenser may be calculated from formula (35). Generally, the deviations from these simple conditions give rise to important corrections, which, at high frequencies, may be larger than the quantity to be measured. While these effects are eliminated in the *comparison* of the power factors of two condensers, their values must be quite accurately calculated if reliable values of the angle θ are to be independently obtained.

If Q_p = observed resistance of the Q arm with alternating current
Q_0 = observed resistance of the Q arm with direct current
ΔW = increase in the resistance of the inductance with alternating current, over its resistance with direct current
$\Delta_r Q$ = change (+ or −) in the resistance of the Q arm with alternating current, due to the residual inductances or capacities of the resistance coils of the bridge
$\Delta Q_0 = Q_p - Q_0$ = measured change in the resistance of the Q arm.

Then, from (35), remembering that $Q_0 = S$, we have

$$\Delta Q_0 = pL \tan \theta + \Delta_r Q \qquad (42)$$

The observations are, however, made on the resistances Q_p' and Q_0', which are included in the Q arm with the inductance.

If W_p = resistance of the coil with alternating current
W_0 = resistance with direct current

and $\therefore \Delta W = W_p - W_0$, then, putting $\Delta Q_0' = Q_p' - Q_0'$

$$Q_p = Q_p' + W_p$$
$$Q_0 = Q_0' + W_0$$
$$\therefore \Delta Q_0 = \Delta Q_0' + \Delta W$$

also $$\Delta_r Q' = \Delta_r Q.$$

Substituting these relations in (42)

$$\Delta Q_0' + \Delta W = pL \tan\theta + \Delta_r Q' \qquad (42a)$$

or

$$\tan\theta = \frac{\Delta Q_0' + (\Delta W - \Delta_r Q')}{pL} \qquad (43)$$

where $\Delta Q_0'$ is the measured change in the resistance, exclusive of the coil itself.

If l_1, l_2, l_3, l_4, l_5 are the residual inductances or capacities of the P, Q, R, S, r arms of the bridge, respectively, then [13]

$$\begin{aligned} PS - RQ = {} & p^2[l_1 l_4 - l_3(l_2 + L)] \\ & + p_2 C\,[PRl_4 + SR(l_1 + l_5) + PS(l_3 + l_5) \\ & \qquad + r(Sl_1 + Sl_3 + Pl_4 + Rl_4)] \\ & - p^4 C(l_1 l_3 + l_3 l_5 + l_5 l_1)\, l_4 \end{aligned}$$

When $P = R$, this reduces to

$$\begin{aligned} \Delta_r Q_0 = {} & \frac{p^2}{P}[(l_2 l_3 - l_1 l_4) + l_3 L] \\ & - p^2 C\left[S\left(1 + \frac{r}{P}\right)(l_1 + l_3) + l_4(2r + P) + 2Sl_5\right] \\ & + \frac{p^4 C}{P}(l_1 l_3 + l_3 l_5 + l_5 l_1)\, l_4 \end{aligned}$$

Usually the expression $(l_2 l_3 - l_1 l_4)$ is very small, and may be neglected with respect to $l_3 L$; also terms of the third degree in l_1, l_2, etc., need not be considered. The following more simple formula is, therefore, obtained:

$$\Delta_r Q_0 = \frac{p^2 L l_3}{P} - p^2 C\left[S\left(1 + \frac{r}{P}\right)(l_1 + l_3) + l_4(2r + P) + 2Sl_5\right] \qquad (44)$$

It is not easy to obtain accurately, ΔW, the change of resistance with the frequency. The resistance of standards of inductance of the larger values, wound with unstranded wire as large as No. 24 B. and S., is appreciably different, even at 100 cycles, from the steady current value. The increase of resistance of a 1-henry coil wound with wire of about this size was found to be 0.02 ohm, or two ten-

[13] Rosa and Grover, this Bulletin, **1**, p. 307 (equation 15); 1905.

thousandths of the resistance of the coil. This is about one-tenth of the calculated change (at 100 cycles) of the resistance Q, due to the power factor of the standard condenser, an inductance of 1 henry being used in the bridge. ΔW can be made smaller by using lower frequencies in the measurements, and by stranding the wire.

If the frequency is reduced, then by (35) the value of L must be increased to give the same value of ΔQ, and increasing L, other things being equal, increases the total resistance of the coil used. But since the effect of the frequency increases as the square, while the value of ΔQ in (35) is proportional to the first power, it is possible to decrease the value of ΔW very considerably without unduly decreasing ΔQ.

ΔW depends, however, on the electrostatic capacity of the coil itself, and on the capacity of the leads used. The increase of resistance, due to these causes, is proportional to the capacity, to the square of the frequency, and to the inductance of the coil, and although smaller than the effect of eddy currents, is, in the coil mentioned above, appreciable. It seems, therefore, impracticable to make ΔW negligible, and its value must be determined and allowed for. This may be done approximately by one of the following methods:

(1) By assuming the value of θ, determined by stepping up from the air condenser, and by working at a higher frequency, where the term $pL \tan \theta$ is not so large as $\Delta_r Q'$ (see equations 42*a* and 44) an approximate value of ΔW is obtained. Reducing this value in the ratio of the square of the frequencies, a value for ΔW at the lower frequency is obtained, which is sufficiently accurate when the correction is small.

(2) We may avoid thus assuming a value for the power factor by making the coil to be tested one of the coils of a mutual induction in Maxwell's method for comparing a mutual induction with the self-induction of one of its coils. In this case the term $pL \tan \theta$ drops out of (42*a*).

(3) Let a coil be wound with stranded wire, with the separate strands insulated, and let the value of ΔW, in this case very small, be found by one of the methods just described. Then this coil may be used as a standard, and the value of ΔW for any other coil of about the same inductance, may be found by comparing the two

coils directly in a bridge, or by balancing them successively against a third coil, both with direct current and with alternating current of the desired frequency.

8. MEASUREMENT OF SMALL CONDENSERS.

In comparing condensers of a few thousandths of a microfarad or less, the error arising from the electrostatic capacity of the bridge becomes relatively so important that direct comparisons by these methods give, at best, only approximate results, and a substitution method must, of necessity, be used.

For these measurements the Parallel Resistance Method is quite inapplicable if correct capacity ratios are desired; the power factor measurements are, however, not thus affected. The conditions are not favorable for using Anderson's method, owing to the insensitiveness of the bridge when large inductances are compared with small capacities. It is possible that better determinations of the power factor by this method might be obtained if the variable resistance r were inserted in the supply circuit, as suggested by Stroud and Oates.[14] The Series Resistance Method gives good results if the corrections given in (13) and (14) be applied. The Series Inductance Method, however, seems most free from objection. The following procedure has been found satisfactory. The condenser to be tested is first balanced against an auxiliary condenser of the same, or nearly the same, nominal value, the balance being obtained by alternately varying the resistance R_3 and the inductance L_3. Then, without changing the resistances, R_3, R_4, a calibrated variable air condenser of suitable range is substituted for the condenser to be tested, and a balance is again obtained by varying the capacity of the standard and the inductance in L_3. The required capacity is then equal to the capacity corresponding to the setting of the air condenser, and the power factor is calculated from the observed values of the inductance by equation (21), the phase difference, θ_1'', of the air condenser being taken equal to zero. It sometimes happens that the inductance necessary for balance is beyond the range of the variable. In this case coils of fixed and convenient values will be inserted. When, now, the air condenser is inserted,

[14] Phil. Mag., **6**, p. 707; 1903.

these must be removed and R_3 must be increased by an amount equal to the resistance subtracted

The whole process may be accelerated, by using for the auxiliary condenser, C_2, another variable air condenser. Then, making R_3 and R_4 nearly equal, the bridge is balanced, first with the condenser to be tested, by varying the capacity C_2 and the inductance L_3; second, with the standard air condenser (the capacity of C_2 being left unchanged) by varying the capacity of the standard and the inductance of L_3, having first compensated any change in R_3 due to the removal of inductances.

The advantage arising from the use of a *variable standard* lies not so much in the quickness with which the balance may be found, but in the fact that nothing is changed in the bridge, when one condenser is substituted for the other, except the value of the inductance in the ratio arms. This insures that the electrostatic effects, which are so important, are as nearly as possible the same in the two cases, and are, therefore, without any considerable effect on the result.

The method just described may be used with advantage in the testing of cables. If the cable is measured immersed, care must be taken that the resistance of the water (which is in series with the capacity of the cable) is not so large as to cause an appreciable error in the measurement of the power factor. Of course, with such small capacities, a resistance of 50 or 100 ohms is not very important, but, in any case, the resistance should be measured and a correction applied if necessary.

9. THE APPARATUS.

The resistances R_3 and R_4 were ordinarily about 1,000 or 2,000 ohms. It is convenient to use for the adjustable part of R_3 a dial box reading to 0.1 ohm directly. The closer adjustment, necessary when capacities of 0.1 mf and greater were compared, was obtained by using a slide wire of 0.14 ohm resistance, which allowed direct readings to 0.001 ohm to be made. With 1 mf in each condenser arm, a change in R_3 of 0.02 ohm in a total of 1,000 ohms could be detected.

The variable inductance (of the Ayrton-Perry type, constructed by the Leeds & Northrup Company) had three different ranges, viz:

0.5 to 3 millihenry,
4.0 to 24 millihenry,
5.0 to 42 millihenry.

The middle range was ordinarily used when two condensers of nearly equal power factor were to be compared at a frequency of 100. For a frequency of 1,000 cycles the smallest range was employed. A fixed coil of 5 to 10 millihenrys, or less, was inserted in the arm adjacent to the auxiliary condenser, to avoid settings below the range of the variable inductance. In the comparison of condensers of 1 microfarad capacity, the inductance could easily be set to 0.01 millihenry. For making this fine adjustment a slow-motion attachment was found very convenient.

The current for measurements at 30 to 100 cycles was taken from the alternator of a small motor-generator set driven from a storage battery. This machine is rated at 2.5 amperes at 120 volts and 120 cycles. The electromotive force is far from being sinusoidal.

For higher frequencies up to 1,000 cycles, the various machines of a harmonic alternator set, constructed especially for the Bureau of Standards by the General Electric Company, was used. From this set frequencies, which are the odd multiples of 60 cycles up to 900 cycles, are available, and by increasing the speed of the driving motor, 1,000 cycles can be obtained. These machines generate electromotive forces which approximate to sine waves.

In the work at the lower frequencies, and for the most part at the higher frequencies also, vibration galvanometers were used as detectors. The galvanometers for frequencies up to 200 cycles were of the Rubens[15] form, made by W. Oehmke, of Berlin, while the instruments responding to the higher frequencies were of the Wien[16] type, constructed by E. Feldhausen, Aachen.

There is little to choose between the two forms, both being sensitive and satisfactory.

The frequency for which the galvanometer suspension was tuned was determined and maintained by an auxiliary bridge, as described in a previous paper.[17] A condenser of known capacity, placed in

[15] Wied. Annalen, **56**, p. 27; 1895.
[16] Wied. Annalen, **4**, p. 425; 1901.
[17] Rosa and Grover, this Bulletin, **1**, p. 298; 1905.

the fourth arm of a Wheatstone bridge, is charged and discharged by a rotating commutator coupled to the shaft of the alternator. From the known capacity of the condenser and the resistances of the bridge, the frequency at which the bridge is balanced can be calculated. To determine the frequency with which the galvanometer suspension is in resonance, the galvanometer is inserted in a Wheatstone bridge supplied with current from the alternator. The balance of this bridge having been slightly disturbed, the frequency is adjusted by a resistance in the armature circuit of the motor until the deflection of the galvanometer is a maximum. At the higher frequencies, where the machine was inaccessible, an ammeter in the field circuit of the alternator gave a fairly satisfactory indication of the point where the proper speed was attained, the adjustment being made by a resistance in the field circuit.

The advantages connected with the use of a vibration galvanometer, which are discussed on pages 296 and 297 of the article just referred to,[17] are especially emphasized here. The value of the power factor of a poor condenser is extremely sensitive to changes of frequency, as has been pointed out by Monasch, and, further, the effect of each harmonic in the electromotive force wave is magnified in the condenser current in proportion to the frequency of the harmonic. It is therefore impossible to make satisfactory setting with a telephone unless the overtones are so weak as not to confuse the balance point corresponding to the fundamental. This condition is pretty well realized with the high-frequency currents used in these experiments, and it was found possible to make fairly good settings with a telephone. To do so, however, it was necessary to eliminate a very important disturbance. It was at first found impossible, by any change of the ratio arms and the phase compensation, to bring the current due to the fundamental to zero, and the settings of the bridge were not only uncertain but inaccurate. It was finally discovered that by touching either terminal of the telephone circuit with the hand the resistance R_3 and the phase compensation could be so adjusted that all sound of the frequency of the fundamental disappeared, and only very high whistling noises due to the small harmonics remained, which were too weak to vitiate the determination of the balance. However, the setting obtained by touching one

[17] Rosa and Grover, this Bulletin, **1**, p. 298; 1905.

terminal was different from that found when the other terminal was touched, both in the value of the capacity ratio and in the value of the power factor. The differences with capacities of 0.1 mf were as large as 1 in 1,000 in the capacity ratio, and much more in the apparent value of the power factor. Experiment, however, showed that the values found by this procedure, using the substitution method, were in agreement with those obtained with the vibration galvanometer, and that the latter values were not sensibly different from the means of the two settings taken, touching the two terminals. Laying the hand upon one terminal, therefore, seems to shift the balance in one direction from the correct point as much as touching the other terminal shifts the balance in the other direction.

The following explanation of these facts seems to be in accord with the observations. The telephone consisted of two watch-case receivers, held against the ears by metal straps passing around the head of the observer. Each telephone winding, being insulated from the case, was at the potential of the middle of the bridge, while the case and the head of the observer were at a very different potential, and had, therefore, an appreciable capacity with respect to the telephone winding. The sound in the telephone which could not be removed by adjusting the bridge is, according to this hypothesis, due to the current which charges and discharges this capacity.

When, now, either junction of the telephone circuit and the bridge is touched by the hand, the observer and the case of the telephone are brought to the potential of the middle of the bridge, and no charging current flows. Touching this junction has, nevertheless, the effect of changing the electrostatic capacity of the bridge with respect to its surroundings, and thus shifts the balance point. When the hand is laid on the other terminal, the displacement of the balance point is, by symmetry, of sensibly the same value, but in the opposite direction.

The vibration galvanometer is free from this source of error. Since, in addition to those advantages already discussed, it is much more sensitive than the telephone at low frequencies, and about its equal at 1,000 cycles, the use of the vibration galvanometer, instead of a telephone, is to be recommended. To the author the former is much the pleasanter instrument to use, and when a large number of

settings are to be made it is much easier to maintain the initial precision of the settings than with the telephone.

A number of alternating-current galvanometers have been suggested in recent years, most of which consist, essentially, of a coil which carries the small current to be measured, suspended in the field of an electro-magnet, which is excited from the same supply as the current in the coil. There is, in general, an appreciable phase difference between the current in the field and in the coil, and when the galvanometer is used as a zero instrument in a bridge it is not feasible to keep this phase angle at a constant value. From page 379 and from Fig. 7 it is easy to see that when the bridge is nearly balanced the phase of the galvanometer current is very sensitive to changes in the bridge arms, and may vary through a wide angle. Such a galvanometer will give zero deflection whenever the phase difference of the galvanometer current and the current in the field is 90°, irrespective of the actual value of the galvanometer current; and since balance is reached by alternately adjusting two variables in the bridge, a large number of such points will be found. Further, such an alternating current galvanometer will give zero deflection when the *integral* current through its coil is zero, even though the instantaneous current may depart considerably from zero during the cycle. Such an instrument is, therefore, unfitted for a zero instrument when the conditions for balance presuppose that the galvanometer current is at every moment actually zero. On the contrary, both the vibration galvanometer and the telephone fulfill this requirement, since their indications depend only on the *amplitude* of the instantaneous current.

10. COMPARISON OF THE METHODS.

The foregoing methods have been used during the past two years and a half to measure the power factor of both mica and paraffined paper condensers by a number of the well-known manufacturers of England, France, Germany, and America. From the large mass of data thus accumulated it seems advisable to present only such results as shall suffice to illustrate the practical use of the methods, and to indicate the order of magnitude of the power factors which may be expected in condensers of different kinds.

In Tables I to VII, inclusive, are given the results of tests made to determine how closely the different methods can be relied upon

TABLE I.

Series Resistance Method.

March 15, 1907.

R_4=1,100 and 1,600 ohms. n=99.42 cycles per sec. Capacity of B=1.0064.

1	3	4	5	6	7	8	9	10	11
C_1	R_3	r_1	r_2	$\frac{C_1'}{C_1''}=\frac{R_3''}{R_3'}$	C_1' (mean)	pC_1r_1	$\tan(\theta_1'-\theta_1'')$	$(\theta_1'-\theta_1'')$	Temp.
	ohms	*ohms*	*ohms*		*mf*			′ ″	°
SH	1,075.135	14.47	30	1.02199		0.009298	0.00949	32 38	
B	1,098.78	29.89			1.02852	.018792			18.1
SH	1,563.57	14.45	30	1.02199		.009286	.00949	32 38	
B	1,597.96	29.87				.018780			
Mp	1,117.43	7.66	10	0.98330		.004733	.001516	5 13	
B	1,098.78	9.94			0.98958	.006249			18.1
Mp	1,625.12	7.62	10	0.98329		.004707	.001530	5 14	
B	1,597.97	9.92				.006237			
Wa	1,075.03	2.03	10	1.02206	1.02860	.001304	.004914	16 54	20.5
L	1,094.12	8.85		1.00422	1.01064	.005586	.000632	2 10.5	20.45
B	1,098.74	9.89				.006218			
Wa	1,563.37	1.99	10	1.02210		.001280	.004916	16 54	
L	1,591.19	8.80		1.00422		.005586	.000640	2 12	
B	1,597.91	9.86				.006196			

to give the same values for the capacity and power factor of a given condenser. Each method was compared directly with the Series Inductance Method at two frequencies. Substitution methods only were used. In Table I is given in some detail, the measurements and calculation of the capacity and power factor of four condensers by the Series Resistance Method (equations 15 and 16). Column 3 shows the observed values of R_3' and R_3'', and column 4 the corresponding values of the resistances r_1' and r_1'' in series with the condensers C_1' and C_1'' when the bridge was balanced. Since these condensers were compared by substitution the values of C_2 and r_2 do not have to be known. In column 6 is calculated the ratio of the capacities to that of the standard condenser B. From the known capacity and temperature coefficient of this condenser, the capacities

of the other condensers could then be immediately calculated and are given in column 7. Column 8 contains the calculated values of $pC_1'r_1'$ and $pC_1''r_1''$, the difference of which (equation 16) is the tangent of the difference between the phase angles of the condensers and that of the standard, column 9.

In Table II are given measurements on the same condensers on the same day and at the same frequency, the Series Inductance

TABLE II.

Series Inductance Method.

March 15, 1907.

R_4=1,102.8 and 1,602.8 ohms. Capacity of B=1.0064 mf.

1	3	4	5	6	7	8	9	10	11
C_1	R_3	L_3	L_4	$\frac{C_1'}{C_1''}=\frac{R_3''}{R_3'}$	C_1' (mean)	$\frac{pL_3}{R_3}$	$\tan(\theta_1'-\theta_1'')$	$(\theta_1'-\theta_1'')$	Temp.
	ohms	*millihenrys*	*millihenrys*		*mf*			′ ″	°
SH	1,075.655	26.42	10	1.02208		0.015343	0.00949	32 37	
B	1,099.415	10.30			1.02860	.005853			18.1
SH	1,564.11	34.00		1.02206		.013579	.00959	32 58	
B	1,598.61	10.44				.003987			
Mp	1,120.31	13.115	10	0.98331	0.98959	.007313	.001520	5 13	18.1
Wa	1,077.81	18.46		1.02208	1.02860	.010700	.004907	16 52	20.5
L	1,096.97	11.30		1.00423	1.01064	.006435	.000642	2 12	20.45
B	1,101.615					.005793			
Mp	1,627.995	14.475	10	0.98330		.005554	.001525	5 14.5	
Wa	1,566.25	19.89		1.02206		.007933	.004904	16 51.5	
L	1,594.07	11.865		1.00422		.004650	.000621	2 8	
B	1,600.805					.004029			

Method being used. The comparisons were made by substitution as in the preceding table, and in columns 8 and 9 are given the results of the calculation of the phase angle differences by formula (21). The capacities are calculated by the simple formula (3).

The measurements of Tables I and II together with those made on the same day at a lower frequency are collected in Table III. In column 4 are given the capacities calculated by formula (3) and in column 5 the same values corrected by using (22.) It will be seen that in the case of the condensers with the smaller power factors this correction is less than one part in 100,000, but reaches four parts in

100,000 with the condenser, SH. The capacities found by the two methods are in good agreement, the greatest difference being three parts in 100,000. From columns 7 and 8 it may be seen that very nearly the same values of $(\theta_1' - \theta_1'')$ are derived by the two methods. In column 9 the means of the two values of $(\theta_1' - \theta_1'')$ are corrected for the resistance of the leads used with the condensers, as discussed on page 400. The phase angles θ_1 of the various condensers tested follow at once from the known values of the power factor of the standard and the resistance of its leads. These values appear in column 10.

TABLE III.

Comparison of Results by the Series Resistance and Series Inductance Methods.

Value of θ for B with leads used was at 50 cycles 0° 1′ 30″.
" " " " " " " " " " " 100 " 1′ 15″.

1	2	3	4	5	6	7	8	9	10
Date	Condenser	Frequency	Series Inductance: Observed Capacity by (3)	Series Inductance: Observed Capacity by (22)	Series Resistance: Observed Capacity	$(\theta_1' - \theta_1'')$: Series Inductance	$(\theta_1' - \theta_1'')$: Series Resistance	Mean, corrected for leads	θ_1
1907.			*mf*	*mf*	*mf*	′ ″	′ ″	′ ″	′ ″
Feb. 15	E	53.17	0.33842	0.33842	0.33844	− 0 24	− 0 25	− 0 25	0 58
		99.75	0.33836	0.33836	0.33836	− 0 10	− 0 3	− 0 7	1 0
	No.5478	53.17	0.50656	0.50656	0.50656	+ 0 46	+ 0 47	+ 0 45.5	2 17
		99.75	0.50640	0.50640	0.50640	+ 0 41.5	+ 0 42.5	0 41	1 55
	Wa	53.17	1.03252	1.03251	1.03251	15 0	15 6	15 3	16 33
		99.75	1.03076	1.03074	1.03077	17 0.5	17 5.5	17 3	18 18
Mar. 15	SH	52.48	1.03268	1.03265	1.03260	40 24	40 37	40 29.5	42 0
		99.42	1.02860	1.02856	1.02852	32 47	32 38	32 39	33 54
	Wa	52.48	1.03033	1.03032	1.03028	14 29	14 16	14 22.5	15 52
		99.42	1.02860	1.02858	1.02860	16 52	16 54	16 53	18 8
	Mp	52.48	0.99037	0.99037	0.99037	4 53	4 56	4 53.5	6 24
		99.42	0.98959	0.98959	0.98958	5 14	5 13.5	5 12	6 27
	L	52.48	1.01108	1.01108	1.01108	2 22.5	.....	2 21	3 51
		99.42	1.01064	1.01064	1.01064	2 10	2 11	2 7.5	3 22

Table IV serves to illustrate the use of the Parallel Resistance Method. For convenience a resistance R_2 of 100,000 ohms was

TABLE IV.

Parallel Resistance Method.

March 5, 1907.

Capacity of B 0.5 at 53 cycles, 0.50443 mf B at 53 cycles, 1.0067 mf.
" " " " " 100 " 0.50434 " " " " " 1.0065 "

1	2	3	4	5	6	7	8	9	10	11
C_1	C_2	R_1	r_1	r_2	$\frac{C_1'}{C_1''}=\frac{R_1''}{R_1'}$	C_1' by (25)	C_1' by (27)	$(\theta_1'-\theta_1'')$ calculated	Frequency	Temp.
		ohms	*ohms*	*ohms*		*mf*	*mf*	′ ″		°
L	X_6	1,094.195	102,220	100,000	1.00446	1.01120	1.01120	2 22	53.43	19.95
B		1,099.075	100,290							19.5
L		1,591.32	102,250		1.00448			2 22		
B		1,598.42	100,320							
L 0.5	X_6'	1,097.08	101,330	100,000	1.00047	0.50467	0.50467	2 44.5	53.43	19.7
B 0.5		1,097.60	100,010							
L 0.5		1,595.475	101,330		1.00048			2 44.5		
B 0.5		1,596.25	100,010							
Wa	X_6	1,074.25	114,970	100,000	1.02312	1.02999	1.03001	14 55	53.43	19.5
B		1,099.09	100,320							
Wa		1,562.265	115,040		1.02314			14 59		
B		1,598.425	100,310							
SH	X_6	1,071.89	160,350	100,000	1.02536	1.03224	1.03233	39 33	53.43	17.3
B		1,099.08	100,290							
SH		1,558.86	160,350		1.02537			39 33		
B		1,598.42	100,320							
L	X_6	1,093.705	103,630	100,000	1.00409	1.01064	1.01064	2 12	95.25	19.95
Wa		1,075.03	138,750		1.02153	1.02819	1.02847	16 47		19.5
SH		1,074.935	228,400		1.02162	1.02829	1.02865	32 31		17.3
B		1,098.175	100,080							19.5
L		1,590.595	103,700		1.00409			2 8		
Wa		1,563.435	138,910		1.02153			16 42		
SH		1,563.28	228,800		1.02163			32 32		
B		1,597.10	100,150							
L 0.5	X_6'	1,596.27	102,200	100,000	1.00012	0.50440	0.50440	2 27	95.25	
B 0.5		1,596.46	100,000							

placed in parallel with the auxiliary condenser C_2 so that the resistances R_1 are of the same order of magnitude. The capacities calculated by the usual formula (25) and given in column 7 require correction owing to the appreciable capacity of these large resistances. Since these have been determined for a few of the resistance coils only, the corrections, which are calculated by (27), can be made only

approximately. It will be seen, however, from column 8 that they are by no means negligible with the condensers having the larger power factors.

From Table V it is evident that when this correction is negligible, as is the case with the condensers L and L 0.5, the two methods give

TABLE V.

Comparison of Results by the Parallel Resistance and Series Inductance Methods.

March 5, 1907.

Value of θ for B at 53 cycles 1′ 30″
100 " 1′ 15″
" " " " B 0.5 at 53 " 1′ 35″
100 " 1′ 17″

Condenser	Frequency	Capacity		Mean $(\theta_1'-\theta_1'')$		$\theta_1'-\theta_1''$ corrected for leads	θ
		by P. R.	by S. I.	by P. R.	by S. I.		
		mf	*mf*	′ ″	′ ″	′ ″	′ ″
L	53.43	1.01120	1.01119	2 22	2 22	2 21	3 51
	95.25	1.01064	1.01065	2 10	2 9	2 7	3 22
L 0.5	53.43	0.50467	0.50468	2 44.5	2 45	2 44	4 19
	95.25	0.50440	0.50440	2 27	2 27.5	2 25	3 42
Wa	53.43	1.03001	1.03006	14 57	14 54	14 55	16 25
	95.25	1.02847	1.02847	16 45	16 47	16 46	18 1
SH	53.43	1.03233	1.03247	39 33	39 44	39 37	41 7
	95.25	1.02865	1.02882	32 32	32 27	32 28	33 43

substantially the same values for the capacities. Apparently, however, the data from which the corrections to the capacities of Wa and SH have been calculated are not accurate. The larger the correction is the greater is the discrepancy between the values found by the two methods, indicating an appreciable error in the capacities assumed for the resistance coils. The phase differences, however, are in close agreement with those found by the Series Inductance Method.

In Table VI are given results obtained by means of an arrangement of the Anderson bridge found convenient for frequencies of 50 and 100. Since Q' could be set to 0.001 ohm the values of $\Delta Q'$ are not in error by more than 0.002 ohm. The capacities, calculated by the simple formula (36*a*) appear in column 5, and the values corrected

by (36) in column 7. This correction is by no means negligible in the case of SH.

TABLE VI.

Capacities and Phase Differences by Anderson's Method.

February 26, 1907.

P=R=350 ohms. Inductance, Coil C=1.0145 henrys.
S=700 " Capacity of B at 53 cycles=1.00665 mf.
100 " =1.0064 "

1	2	3	4	5	6	7	8	9	10	11
Condenser	Frequency	r	2 r+P	$\frac{C_2}{C_1}$ by (36a)	C_2	C_2 corrected	Q′	ΔQ′	$(\theta_2-\theta_1)$ calculated	Temp.
		ohms	*ohms*	*mf*	*mf*	*mf*	*mean*	*ohms*	′ ″	°
L	53.72	542.125	1,434.250	1.00450	1.01119	1.01119	605.455	0.229	2 18	20.0
B		545.349	1,440.698				605.226			19.85
Wa	53.72	528.535	1,407.070	1.02394	1.03075	1.03077	606.762	1.492	14 58	20.0
B		545.377	1,440.754				605.270			
SH	53.72	527.560	1,405.120	1.02536	1.03218	1.03231	609.119	3.778	37 56	16.5
B		545.375	1,440.750				605.341			
L	98.11	542.457	1,434.914	1.00423	1.01069	1.01069	605.506	0.378	2 5	20.1
B		545.495	1,440.990				605.128			20.1
Wa	98.11	529.632	1,409.264	1.02252	1.02909	1.02912	608.146	3.067	16 51.5	20.0
B		545.496	1,440.992				605.079			
SH	98.11	529.986	1,409.972	1.02200	1.02857	1.02865	610.690	5.620	30 54	16.5
B		545.495	1,440.990				605.070			

Table VII shows that the results by this method are in close agreement with those by the Series Inductance Method. In a few cases the differences are larger than can be explained by the accidental errors of the bridge. It is, however, only with the condensers of larger power factor that these discrepancies occur, and they are to be expected, since with these condensers the capacity changes appreciably with only a very small change in the frequency or temperature. With SH, for instance, a change of one cycle in the frequency means a change of about 1 in 10,000 in the capacity.

From these results it is established that the measured values of the capacity and power factor of a condenser are very closely the same whichever of the four methods is used.

TABLE VII.

Comparison of Results by Anderson's Method and by the Series Inductance Method.

Date	Condenser	Frequency	Capacity		$(\theta_1'-\theta_1'')$		$(\theta_1'-\theta_1'')$ corrected for leads	θ	Temp.
			by A. M.	by S. I.	by A. M.	by S. I.			
1907			*mf*	*mf*	′ ″	′ ″	′ ″	′ ″	°
Feb. 8	Wa	100.66	1.03077	1.03082	16 45	16 47.5	16 47	18 2	21.0
	Wb		1.03170	1.03169	13 33	13 36	13 36	14 51	
	Wc		1.11937	1.11950	16 54	16 44	16 44	17 59	
	Wd		1.11757	1.11763	15 22	15 33	15 33	16 48	
Feb. 28	L	53.72	1.01118	1.01119	2 18	2 17.5	2 17	3 47	20.0
		98.11	1.01069	1.01068	2 5	2 4	2 2	3 17	
	Wa	53.72	1.03077	1.03073	14 58	14 58	14 58	16 28	20.0
		98.11	1.02912	1.02912	16 51.5	16 50	16 51	18 6	
	SH	53.72	1.03231	1.03239	37 56	37 49	37 51	39 21	16.5
		98.11	1.02865	1.02867	30 54	31 2	30 56	32 11	

11. RESULTS OF ABSOLUTE DETERMINATIONS.

Table VIII illustrates the determination of the absolute values of the power factors of the various sections of the condenser B, which was used as a standard in this work. The 0.1 mf section and the two 0.2 mf sections in series were separately compared with the 0.1 mf air condenser of the Bureau. Then it was only necessary to compare with one another the two 0.2 mf sections; and the 0.5 mf section with the 0.2, 0.2′, and 0.1 sections in parallel to obtain the value of θ for any single section, or for any combination of the separate sections.

For, from (41*a*) which is applicable to this case, the observed value of θ for the 0.2 and 0.2′ sections in series is the arithmetical mean of the values θ (0.2), θ (0.2′) of the separate sections, and since we also know, by observation, the small difference of these angles we may easily find their values individually. Having, therefore, obtained θ (0.1) by observation and θ (0.2) and θ (0.2′) by observation and calculation, θ (0.2+0.2′+0.1) follows directly from these separate values by equation (39*a*). From this last obtained value is found by means of the observed difference the value of θ for the 0.5 section. The mean of this value, θ (0.5), and of θ (0.2+0.2′+0.1) will be the value of θ for the whole microfarad.

TABLE VIII.

Absolute Determination of Phase Differences by Stepping Up from Air Condenser.

Values for the various sections of condenser B by the Series Inductance Method.

Date	Condenser	Frequency	L_3	R_3	$\frac{pL_3}{R_3}$	tan $(\theta_1'-\theta_1'')$	$(\theta_1'-\theta_1'')$	$(\theta_1'-\theta_1'')$ corrected for leads	Temp.
1907			*millihenrys*	*ohms*			′ ″	′ ″	°
Jan. 29	0.1	52.49	9.575	1109.79	0.002846	0.000396	1 22	1 22	18.8
	Air		8.22	1106.65	.002450				
"	0.2 and 0.2′ in series	52.49	9.605	1111.52	.002850	.000406	1 24	1 23.5	19.4
			9.40	1615.08	.001920	.000402	1 23		
	Air		8.20	1106.60	.002444				
			7.40	1607.80	.001518				
"	0.2′	52.49	9.345	1111.76	.002772	− .000002	−0 0.5	0 2	19.4
			9.00	1615.50	.001837	+ .000019	+0 4		
	0.2	52.49	9.39	1116.27	.002774				
			8.94	1622.03	.001818				
"	0.5	52.49	7.67	1104.34	.002291	.000055	0 11.5	0 10	19.4
			6.64	1604.71	.001365	.000043	0 9		
	0.2+0.2′ +0.1	52.49	7.52	1109.06	.002236				
			6.41	1611.57	.001312				
Jan. 29	0.1	104.25	9.73	1109.50	.005744	.000344	1 11	1 8	19.6
			9.62	1612.09	.003909	.000317	1 5		
	0.2 and 0.2′ in series	104.25	9.75	1111.31	.005746	.000346	1 11.5	1 11	
			9.70	1614.75	.003935	.000343	1 11		
	Air	104.25	9.12	1106.16	.005400				
			8.815	1607.25	.003592				
"	0.2′	104.25	9.72	1111.41	.005728	.000014	0 3	0 4	19.6
			9.58	1615.14	.004463	.000025	0 5		
	0.2	104.25	9.735	1116.02	.005714				
			9.57	1621.67	.004438				
"	0.5	104.25	9.01	1103.92	.005346	.000032	0 6.5	0 6.5	19.6
			8.57	1604.10	.003499	.000031	0 6.5		
	0.2+0.2′ +0.1	104.25	8.995	1108.61	.005314				
			8.53	1610.92	.003468				
Mar. 18	0.1	970	1.900	1107.37	.010457	.000170	0 35	0 32	19.7
	0.2 and 0.2′ in series		1.9125	1109.23	.010508	.000203	0 42	0 39	
	Air		1.8525	1097.94	.010283				
			1.857	1097.81	.010309				
"	0.2′	970	1.916	1113.96	.010525	.000034	0 7	0 7	19.8
	0.2		1.9175	1109.48	.010491				
"	0.5	970	1.859	1101.25	.010288	− .000018	−0 4	−0 4	19.9
	0.2+0.2′ +0.1	970	1.870	1105.915	.010306				

Table IX illustrates the measurement and calculation of the absolute value of the phase angle of the condenser B (= 1 mf) by

TABLE IX.

Absolute Determination of Phase Difference by Anderson's Method.

P=R=350 ohms. Coils used H=0.9980 henry.
S = 700 ohms. C=1.0145 henry.

Date	Condenser	Frequency	Q′	$\Delta Q'_0$	ΔW	$\Delta_1 Q'$	(5)+(6)−(7)	Coil	θ calculated	θ corrected for leads	Temp.
1907			*ohms*	*mean ohms*	*ohms*	*ohms*	*ohms*		′ ″	′ ″	°
Feb. 26	B	D. C.	605.058	0.148	0.016	0.007	0.157	C	1 34	1 33	19.8
		53.72	605.204								
		D. C.	605.056								
		53.72	605.203								
		D. C.	605.052								
		53.72	605.199								
Feb. 26	B	98.11	605.209	0.192	0.065	0.024	0.233	C	1 17	1 15	19.8
		D. C.	605.012								
		98.11	605.200								
		D. C.	605.003								
		98.11	605.192								
		D. C.	604.998								
		98.11	605.187								
Mar. 16	B	99.75	604.657	0.252	0.015	0.024	0.243	H	1 20	1 18	20.0
		D. C.	604.396								
		99.75	604.638								
		D. C.	604.380								
		99.75	604.626								
Mar. 16	B	99.75	605.103	0.190	0.065	0.024	0.231	C	1 15	1 13	20.0
		D. C.	604.908								
		99.75	605.094								

Anderson's method, equation 43. The change in effective resistance of the coils used in the measurements was determined by the first of the methods described on page 406, and these values are probably correct to less than 0.01 ohm. To determine $\Delta_r Q'$, values for the residual inductances and capacities of the bridge arms were assumed which depend on only a few measurements, and the values for the capacities of the coils in the r arm were not measured, but were assumed as the same as those in a similar dial box. The

following were the values of the residual inductances and capacities used in formula 44:

$$l_1 = l_2 = -11.5 \text{ microhenry}$$
$$l_4 = -29.5 \text{ microhenry}$$
$$l_5 = -15 \text{ microhenry}$$

At the frequency of the measurements of Table IX the effect of the capacity of the leads on the apparent resistance of the coil is negligible, otherwise it would have to be included in the value of ΔW.

In Table X are collected the results of a number of determinations of the phase angles of the various sections of condenser B by stepping up from the air condenser, and also those values which have been found by Anderson's method. It will be seen that the

TABLE X.

Summary of Results of Absolute Determinations of Phase Differences of the Sections of Condenser B.

Date	Method	θ for 0.1 section	θ for 0.2and 0.2′ in series	θ 0.2′ $-\theta$ 0.2	θ 0.5 $-\theta\Sigma$0.5	θ for 0.2	θ for 0.2′	θ for 0.2 + 0.2′ +0.1	θ for 0.5	θ for sum = 1.0 mf	Frequency	Temp.
1907		′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″		°
Jan. 29	Air condenser	1 19	1 22	0 3	0 10	1 20.5	1 23.5	1 21.5	1 31.5	1 26.5	52.5	19.4
"	"	1 8	1 8.5	0 5	0 6	1 6	1 11	1 8.5	1 14.5	1 11.5	102.4	19.5
Feb. 2	"	1 29.5	1 31.5					1 31		1 35	52.7	19.5
"	"	1 14	1 16		0 6			1 15.5	1 21.5	1 18.5	102.2	19.0
Feb. 14	"	1 16	1 16	0 3	0 4.5	1 14.5	1 17.5	1 16	1 20.5	1 18	99.8	20.5
"	"	0 30	0 41	0 2	−0 2.5	0 40	0 42	0 39	0 36.5	0 38	920	
Feb. 20	"	1 9.5	1 13.5					1 12.5	1 18.5	1 15.5	98.9	18.5
"	"	0 33.5	0 42					0 40		0 39	920	
Feb. 26	Anderson's method									1 33	53.7	19.8
"	"									1 15	98.1	
Mar. 16	"									1 15.5	99.7	
" 18	Air condenser	1 10	1 12	−0 2	0 3.5	1 12.5	1 11.5	1 11.5	1 15	1 13	99.8	
"	"	0 34	0 40.5		−0 1.5			0 39	0 37.5	0 38	970	

values obtained at the same frequency, but on different days, differ among themselves by as much as 3″ from the mean value. This is due to the fact that the sensitiveness of the settings, when capacities as small as 0.1 mf are being compared with 100 volts on the bridge,

is not quite large enough to give θ to the nearest second. It is probably safe, however, to use the following values for the power factor of the sum of the sections of condenser B:

Frequency	θ
	′ ″
50	1 30
100	1 15
970	0 38

The determinations by Anderson's method are in pretty good agreement with these values, but since they are affected by the uncertainties in ΔW and $\Delta_r Q'$, they have not been used in finding the means. They are nevertheless of value as a check on the results found with the air condenser.

12. ILLUSTRATION OF METHODS FOR SMALL CAPACITIES.

Tables XI and XII illustrate the methods suggested for the measurement of capacities of a few thousandths of a microfarad. The 0.002 and 0.005 mf condensers, Table XI, were balanced against the mica auxiliary condenser by adjusting R_3 and L_3. With 100 volts on the bridge, the resistance R_3 could not be adjusted better than to the nearest ohm in 1,500, and L_3 could not be set closer than about one-half of a millihenry. A 0.005 mf air condenser was then substituted for the condenser to be tested, and its capacity varied together with L_3 to get a second balance. In the case of the 0.001 mf sections the adjustment was made with still less difficulty. The condenser to be tested was balanced against the 0.005 mf air condenser by varying the capacity of the latter and by adjusting L_3, the equal ratio arms being left untouched. A 0.001 mf air condenser was next substituted, and its capacity varied to find a second balance, the 0.005 mf air condenser being left at the setting just obtained. Results of measurements on the same condensers using the Series Resistance Method, Table XII, are in good agreement with those of Table XI by the Series Inductance Method. On calculating the corrections to be applied to the capacity ratios and the differences of the power factors due to the capacity of the resistance coils r_1', r_1'', introduced into the condenser arms, it was found that they were smaller than the errors of setting of the bridge. Cases will, how-

ever, occur where the corrections as calculated by formulæ (13) and (14) will not be negligible. The smaller the capacity to be measured, the more will this be likely to occur. In such an event,

TABLE XI.

Results on Small Capacities by Series Inductance Method.

Frequency 99.7.

Date	Condenser	Observed capacity	L_3	R_3	ΔL_3	$\theta_1' - \theta_1'' = \theta$
1907		*mf*	*millihenrys*		*millihenrys*	° ′ ″
April 4	M .005		28.6	1158	16.3	0 30 20
			37.3	1684	25.2	32 15
	Air	.004838	12.3	1158		
		.004841	12.1	1684		
"	N .005		13.9	1116	2.6	0 5 1
			15.6	1622	3.3	4 23
	Air	.005010	11.3	1116		
		.005005	12.3	1622		
"	S .005		13.9	1046	3.15	0 6 29
			16.0	1519	3.9	5 32
	Air	.005183	10.75	1046		
		.005188	12.1	1519		
"	S .002′		26.25	1138	4.25	0 8 3
			23.6	1645	5.1	6 41
	Air	.001910	23.6	1138		
		.001910	18.5	1645		
"	S .002		110.3	1550	91.8	2 7 0
	Air	.002030	18.5	1550		
"	M .002′	.002017	36.8	1077	27.3	0 54 35
	Air		9.5	1077		
"	N .002′		32.2	1014	17.1	0 36 20
	Air	.002015	15.1	1014		
	N .002		36.0	1023	20.9	0 44 0
	Air	.001997	15.1	1023		
"	M .001		91.4	1100	65.2	2 7 0
	Air	.001009	26.2			
	N .001		38.0	1100	17.4	0 34 5
	Air	.000998	20.6			
	S .001		26.25	1100	3.65	0 7 10
	Air	.001110	22.6			

unless the capacities of the resistances are known, it will be better to use the Series Inductance Method. As examples of the importance, when measuring such small capacities, of the electrostatic effects due

to the capacity of the bridge with respect to its surroundings, the following may be cited. In Table XI, in the measurement of No.001 and So.001 the inductance associated with the auxiliary air condenser was 5 millihenrys. If, therefore, the bridge was free from electrostatic errors, the value of L_3 when the 0.001 mf air condenser was introduced should also be 5 millihenrys, since in that case C_1 and C_2 are both air condensers.

TABLE XII.

Results on Small Capacities by Series Resistance Method.

Frequency, 99.7.

Date	Condenser	Observed capacity	r_1	Δr_1	$\theta_1' - \theta_1'' = \theta$
1907			*ohms*	*ohms*	° ′ ″
April 5	M .005		17400	3000	0 31 5
	Air	.004812	20400		
	N .005		20900	400	0 4 20
	Air	.004996	21300		
	S .005		20200	600	0 6 40
	Air	.005148	20800		
"	M .002′		13000	12500	0 54 15
	Air	.002015	25500		
	N .002′		7000	19000	1 22 0
	Air	.002005	26000		
	N .002		16000	10000	0 43 0
	Air	.001994	26000		
	S .002′		38500	2000	0 8 10
	Air	.001900	40500		
	S .002		81000	11000	0 45 45
	Air	.001930	92000		
"	M .001		8000	60000	2 10 0
	Air	.001005	68000		
	N .001		11000	16000	0 34 25
	Air	.000997	27000		
	S .001		6000	3000	0 7 10
	Air	.001110	9000		

The observed values were, however, $L_3 = 26.2$ millihenrys in one case and $L_3 = 22.6$ in the other. That is, the apparent difference in phase angle of the two air condensers was in one case about 50′ and in the other 43′.

Similarly in Table XII, when the two air condensers were directly compared, the difference of the resistances r_1 and r_2, instead of being zero, was in one case 13,000 ohms and in the other 11,000 giving

angles of 30′ and 26′, respectively. When, on the other hand, the two air condensers were compared by the substitution method, no difference in their power factors could be detected. The electrostatic errors were, therefore, in this case satisfactorily eliminated. From this we easily see that the only practicable procedure is to compare the condensers by substitution.

Having now shown that the different methods are in good agreement among themselves, and that the power factors of the reference condensers can be accurately determined, examples will be given to show what values of θ may be looked for in such condensers as are met in practice.

13. EXAMPLES OF POWER FACTORS IN MICA AND PAPER CONDENSERS.

Values found for mica condensers at 100 cycles and 20° C. are given in Table XIII. From these it is evident that for a good mica

TABLE XIII.

Examples of Phase Differences in Mica Condensers.

Frequency 100 cycles. Temperature 20°.

Condenser	Nominal Capacity	θ	Condenser	Nominal Capacity	θ	Condenser	Nominal Capacity	θ
		° ′ ″			° ′ ″			° ′ ″
E	⅓	0 0 30	M_1	⅓	0 3 51	MM_3	0.001	0 17 40
X_{6r}	0.5	0 0 50	W_4	0.1	0 4 30	NN_2	.001	0 18 0
Q	.5	0 1 0	M_2	⅓	0 4 33	FE	.2	0 20 15
X_{6l}	.5	0 1 2	N	.005	0 4 40	MM_4	.001	0 21 50
Q	.2′	0 1 12	L	.05′	0 5 0	NN_3	.001	0 23 20
B	.5	0 1 15	W_5	.1	0 5 40	H	.2	0 24 50
LL	.1′	0 1 24	H	.2	0 5 57	FE	.2	0 30 30
Q	.05	0 1 32	LL	.005	0 6 15	M	.005	0 31 20
W_1	.1	0 1 53	MM_1	.001	0 7 20	N	.001	0 34 15
X_{43}	.5	0 2 3	H	.2′	0 7 20	N	.002	0 44 0
E	.1	0 2 25	L	.05	0 8 0	M	.002	0 54 30
LL	.05	0 2 29	N	.002′	0 8 0	MM_5	.001	1 10 0
Ct	.5	0 2 31	H	.2″	0 9 10	NN_4	.001	1 30 0
W_2	.1	0 2 38	MM_2	.001	0 10 45	MM_6	.001	2 10 0
W_3	.1	0 3 8	FE	.005	0 11 40	NN_5	.001	3 30 0
X_{8r}	.5	0 3 16	NN_1	.001	0 12 30	P_1	.001	5 30 0
X_{41}	.5	0 3 20	LL	.002′	0 14 35	P_2	.001	11 30 0
L	.5	0 3 42	H	.1	0 16 30	P_3	.001	32 15 0

condenser the angle θ lies between about 0′ 30″ and 5′. The larger values in the table were inserted to show how poor mica con-

densers by well-known makers may occasionally be. It is noticeable that the very largest values of the phase difference occur in condensers of small capacity. The small condensers which have been tested are few in number and confined to a very limited number of makers. Of this number, however, only one condenser of less than 0.01 mf was found with a phase difference of less than 5′.

Only a few silvered mica condensers have been measured. Of these the sections marked W in Table XIII are typical. They exhibit no superiority, as far as power factor is concerned, over the ordinary tinfoil and mica condensers. In testing subdivided condensers one is very often struck by the great lack of uniformity in the quality of the various sections. For instance, in one box of six sections, ranging from 0.05 mf to 0.5 mf, values ranging from 4′ to 30′ were found; in another, values from 5′ to 25′; while Tables XI and XII show how extremely large this variation may be in the case of the very small sections of such a condenser. This variability seems to be characteristic of paper condensers also.

In Table XIV those condensers which are marked W, T, and t are telephone condensers. It is among condensers made for this

TABLE XIV.

Examples of Phase Differences in Paper Condensers.

Frequency 100 cycles. Temperature 20°.

Condenser	Nominal capacity	θ	Condenser	Nominal capacity	θ	Condenser	Nominal capacity	θ
	mf	° ′ ″		*mf*	° ′ ″		*mf*	° ′ ″
Mp	1.0	0 6 30	SH	0.5′	0 24 0	t_{19}	2.0	2 20 0
S_{10}	3.0	0 7 58	SH	.5	0 33 0	t_{17}	2.0	3 30 0
S_3	1.7	0 11 0	Be_{34}	.6	0 33 15	t_7	0.6	4 0 0
Wb	1.0	0 14 50	Be_{15}	.6	0 41 10	t_1	.6	4 45 0
Wa	1.0	0 18 5	S_6	3.2	0 44 30	t_6	.6	7 50 0
Lp	1.0	0 19 45	T_2	2.2	1 5 0	t_3	.6	8 30 0
Cp	0.5	0 20 20	t_{18}	2.0	1 30 0			

class of work that the largest power factors have been found. Excepting the condensers W, included in this table, none of the telephone condensers thus far tested have been found with a value of θ below 45′. So great is the power factor of some of these poor paper condensers that the testing current appreciably heats them, and

this rise of temperature, owing to the large temperature coefficient of both the capacity and the power factor, causes the balance of the bridge to change, progressively, so rapidly that snap readings have to be made. The condensers Be have a dielectric of beeswax and rosin (all the rest of the condensers in Table XIV are of paraffined paper). As these are the only condensers of this kind so far tested no general conclusions can be drawn as to the quality of such a dielectric.

14. CONNECTION BETWEEN THE POWER FACTOR AND THE MAGNITUDE OF ABSORPTION EFFECTS.

In Tables XV and XVI the results of some tests with steady current are given in order to show how closely the magnitude of the absorption effects observed in ballistic galvanometer measurements

TABLE XV.

Comparison of Results with Steady Current with Value of Power Factor.

1	2	3	4	5	6	7	8
Condenser	Nominal capacity	θ	Time of charge	Time of discharge	Relative capacity	Residual charges	
						1 sec. charge	10 sec. charge
	mf	° ′ ″	*sec.*	*sec.*		*per cent*	*per cent*
B	0.5	0 1 18	0.1	0.1	1.0016	0.32	0.71
			1.0	8.5	1.0073		
B	0.2′	0 1 10	0.1	0.1	1.0019	.27	.62
			1.0	8.5	1.0066		
B	0.2′+0.2+0.1	0 1 12	0.1	0.1	1.0020	.28	.65
			1.0	8.5	1.0060		
Q	0.5	0 1 0	0.1	0.1	1.0017	.22	.56
			1.0	8.5	1.0054		
Q	0.2′	0 2 51	0.1	0.1	1.0043	.37	.96
			1.0	8.5	1.0100		
X_{6r}	0.5	0 0 50	1.0	8.5	1.0056	.21	
X_{41}	0.5	0 3 20	1.0	8.5	1.0177	.60	
E	0.1	0 2 25	1.0	8.5	1.0082		
X_3	1.0	0 2 40	1.0	8.5	1.0080		

of condensers are connected with the magnitude of the power factor. In column 6 of Table XV are given the relative capacities, with different times of charge and discharge, referred to the capacity with

alternating current at 100 cycles. In columns 7 and 8 are given the sums of the residual charges observed with 1 sec. and 10 sec. charge, respectively, and a certain arbitrary schedule of subsequent successive periods of discharge and insulation. In Table XVI, column 5, is shown the ratio of the capacity with 1 sec. charge to that with 10

TABLE XVI.

Variation of Residual Charge with Value of the Power Factor.

Condenser	Nominal capacity	Dielectric	θ	Ratio of capacities, 1 sec. and 10 sec. charges	Residual charges	
					1 sec. charge	10 sec. charge
	mf		° ′ ″		*per cent*	*per cent*
X_{6r}	0.5	Mica	0 0 50		0.21	
Q	.5	"	0 1 0	0.9976	.22	0.56
B	.2′	"	0 1 10	.9974	.27	.62
X_{6l}	.5	"	0 1 2		.23	
B	.5	"	0 1 18	.9974	.32	.71
Q	.2′	"	0 2 51	.9955	.37	.96
E	.1	"	0 2 25	.9969	.38	.92
Ct	.5	"	0 2 31	.9978		.76
E	⅓	"	0 0 30	.9976		.60
M_1	⅓	"	0 3 51	.9967		1.03
M_2	⅓	"	0 4 33	.9970		0.82
L	.5	"	0 3 42	.9977	.26	.76
L	.05′	"	0 5 0	.9961	.40	1.00
L	.05	"	0 8 0	.9914	1.49	6.93
Mp	1.0	Paraffined paper	0 6 30	.9984	0.25	0.70
S_{10}	3.0	"	0 7 58	.9960	.54	1.62
SH	.5′	"	0 24 0	.9888	1.52	3.70
SH	.5	"	0 33 0	.9848	1.88	4.63
T_2	2.0	"	1 5 0	.8540	11.4	23.8
S	.002′	Mica	0 6 40		2.08	5.38
S	.001	"	0 7 10		1.67	3.44
MM	.01	"	0 33 0		4.88	13.12
NN	.01	"	1 50 0		10.5	26.3

sec. charge, the length of discharge being, in both cases equal to the period of the first swing of the galvanometer, in this case 8½ sec.

From these two tables the conclusion may be drawn that, qualitatively the magnitude of the residual charges and the dependence of the apparent capacity on the time of charge and discharge, both of which depend on the absorption of the condenser, are proportional to the value of the power factor. Cases occur, it is true, where

this seems to be contradicted, but this is most often the case where condensers of nearly equal power factor are being considered. It may be that such cases can be distinguished by something abnormal in the behavior of their power factor with change of frequency or temperature. Ordinarily, therefore, a measurement of the power factor may be depended upon, not only to distinguish between a good condenser and a poor one, but to show which is the better of two condensers of nearly equal grade.

15. SUMMARY.

(1) Four bridge methods (one of them due to Wien) for simultaneously obtaining the ratio of two capacities and the difference of their power factors are described, and the equations for calculating these quantities from the observations are derived.

(2) Various sources of error and methods of eliminating them are discussed. By the use of an auxiliary condenser, two condensers may be compared by substitution. This procedure has the advantage that electrostatic effects are eliminated, and only the relative values of the resistances and inductances of the bridge arms need be accurately determined.

(3) These four methods are compared with one another, and are shown to give results in good agreement. When used as substitution methods, the capacity ratios of condensers of 0.1 mf and greater may be measured to a few parts in 100,000 or better, and a phase difference of only a few seconds of arc, due to a difference in the absorption, may be detected with certainty. The small temperature coefficients of good mica condensers may readily be determined with accuracy.

(4) By the use of a variable air condenser, reliable comparisons of the capacities and power factors of condensers of 0.001 mf or less, such as short lengths of cable, may be easily made.

(5) Methods are given by which the power factor of the condensers to be used as standards may be measured. The Bureau of Standards makes such determinations when requested.

(6) Examples are given to show what values of the power factor may be expected in condensers by leading manufacturers.

(7) A knowledge of the power factor of a condenser gives reliable information as to the order of magnitude of its absorption effects. The determination of the power factor is the best single test of the quality of a condenser which can be made.

Experiments are being made at the Bureau of Standards on the change of the capacity and power factor with changes of temperature and frequency.

I am indebted to Prof. E. B. Rosa for many invaluable suggestions and for the constant interest he has shown in this research.

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